Exploiting forwardness: Satisfiability and Query Entailment in

Forward Guarded Fragment

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TECHNISCHE UNIVERSITÄT DRESDEN

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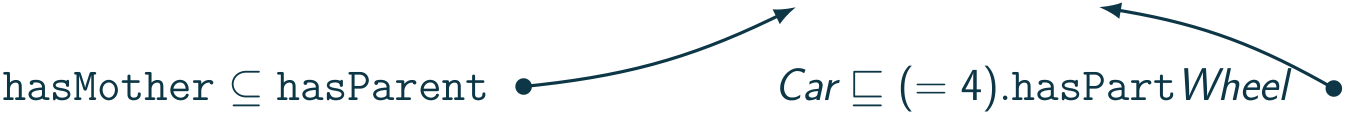

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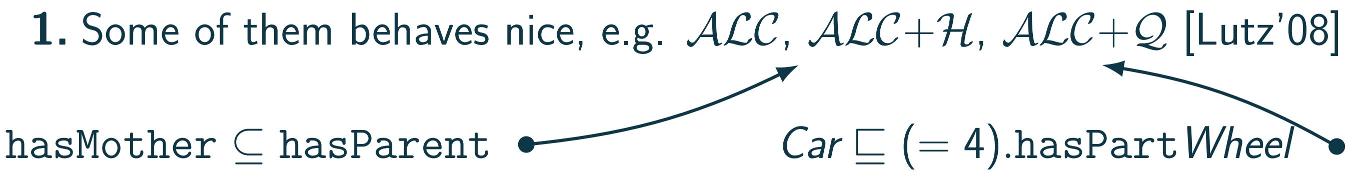

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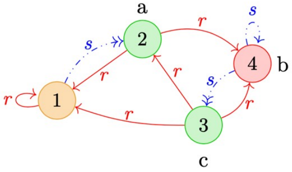

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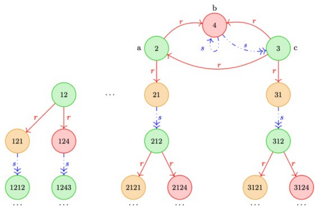

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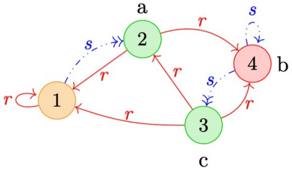

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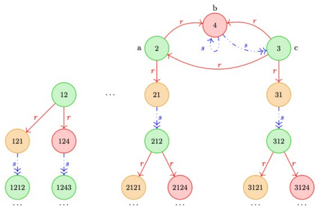

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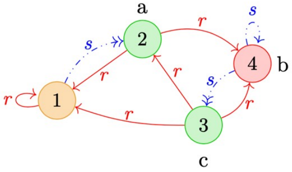

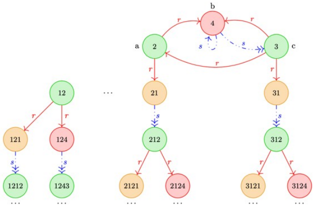

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Yes! $\mathcal{FGF}$ [B. JELIA'21, This talk!]

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Theorem (Bárány et al. 2013)

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Theorem (Pratt-Hartman et al. 2016)

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Lemma (B. 2021)

$\mathcal{FF}$ is reducible to $\mathcal{FL}$ in polynomial time.

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Note that the Forward Guarded Fragment $\mathcal{FGF} := \mathcal{GF} \cap \mathcal{FF}$ also captures $\mathcal{ALC}$.

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Corollary

Data complexity of KB SAT is NP-compl and coNP-compl for querying.

$\mathcal{FGF}$ has FMP and is finitely-controllable.

Two main ingredients: forward-types and HAFs

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A (Σ, n)-*forward type* is a conjunction of atoms with n free-variables $\vec{x}_{1\ldots n}$, which for every relational symbol $\mathrm{R} \in \Sigma$ of arity $\ell = \mathrm{ar}(\mathrm{R}) \leq n$ and every index $1 \leq i \leq n+1-\ell$ contains either $\mathrm{R}(\vec{x}_{i\ldots i+\ell-1})$ or $\neg\mathrm{R}(\vec{x}_{i\ldots i+\ell-1})$.

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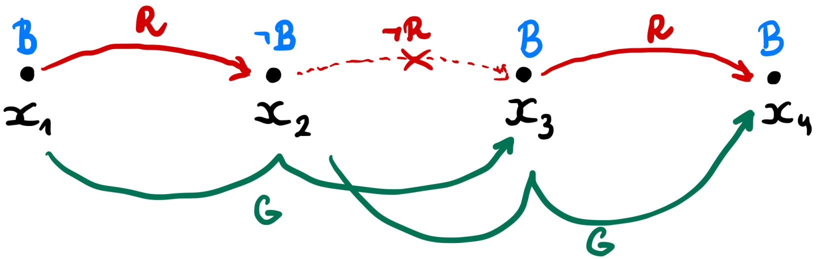
Blue B^1, Red R^2, Green G^3

$(\{R, G, B\}, 4)$-forward tp



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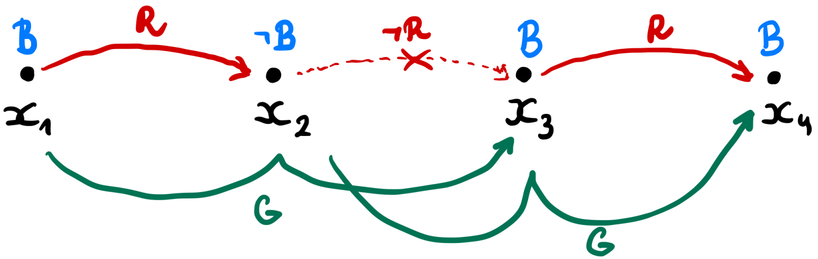

Lemma

The number of different (Σ, n)-types is $\leq 2^{|\Sigma| \cdot n^2}$.

The number of conjuncts in each (Σ, n)-type is $\leq |\Sigma| \cdot n$

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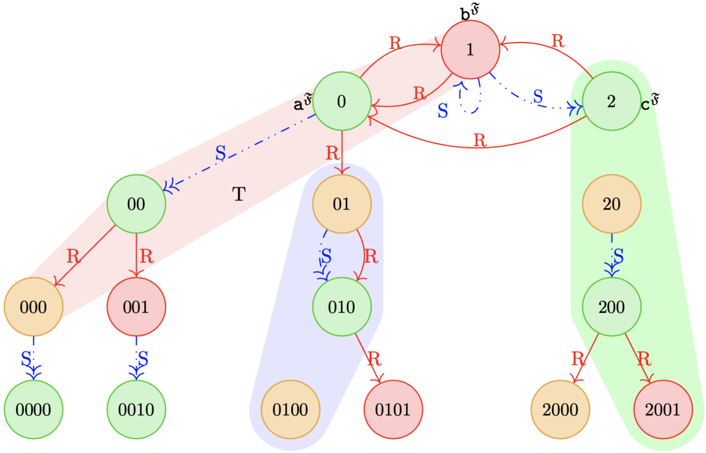

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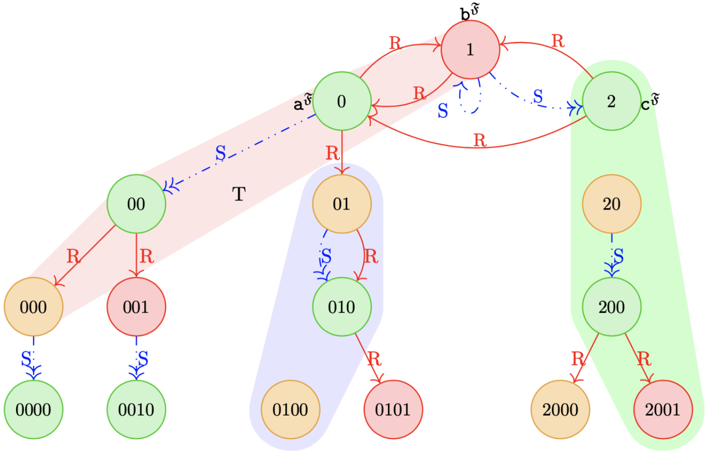

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Every satisfiable $\mathcal{FGF}$ knowledge base has a HAF (counter)model.

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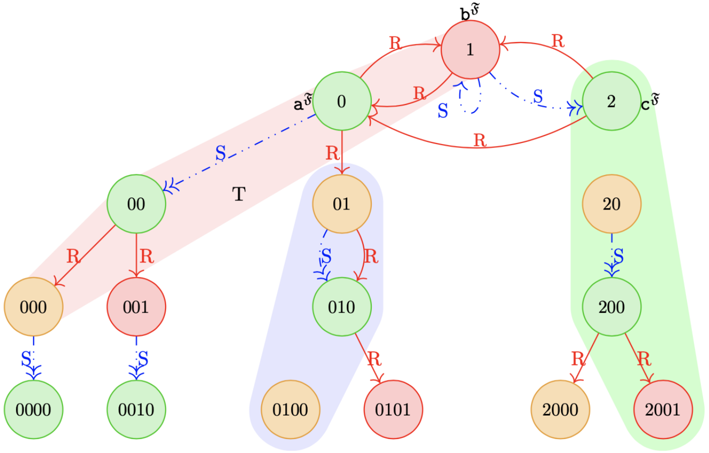

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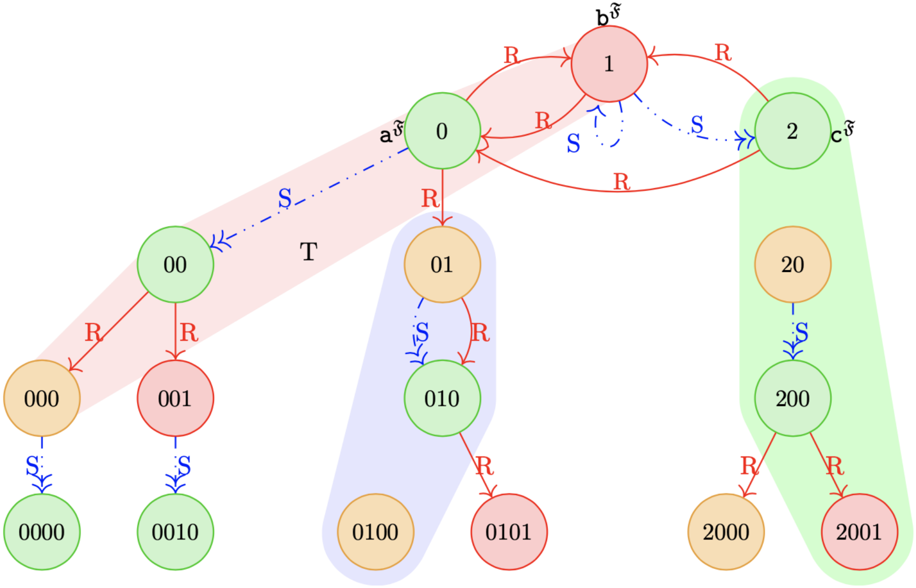

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Thanks for attention!