The Price of Selfishness

Conjunctive Query Entailment for $\mathcal{ALC}_{\mathsf{Self}}$ is 2ExpTime-hard

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Bartosz “Bart” Bednarczyk, Sebastian Rudolph

TU Dresden & University of Wrocław

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Established by the European Commission

Running example: Greek mythology $\mathcal{ALC}_{\mathsf{Self}}$ knowledge base

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Database (ABox)

Knowledge (TBox)

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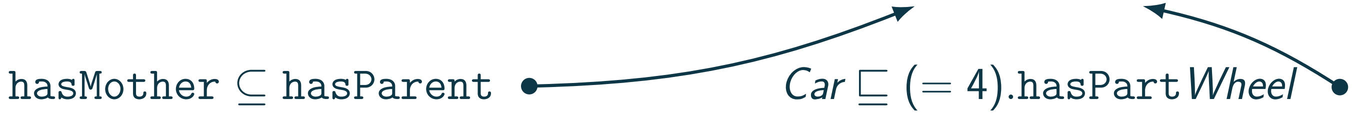

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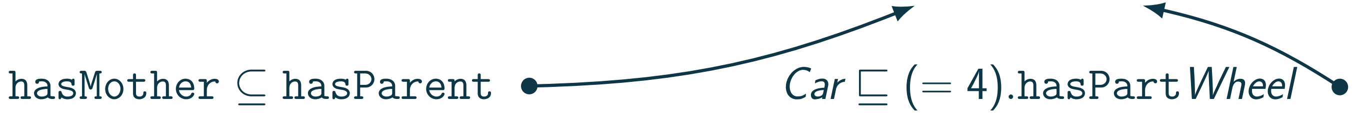

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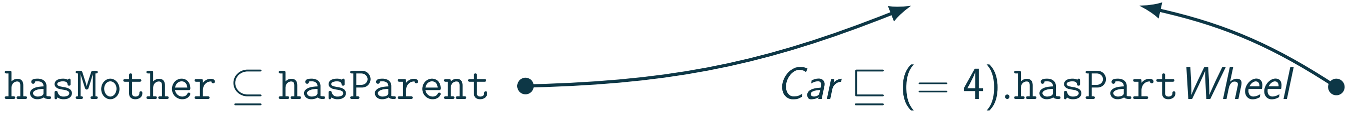

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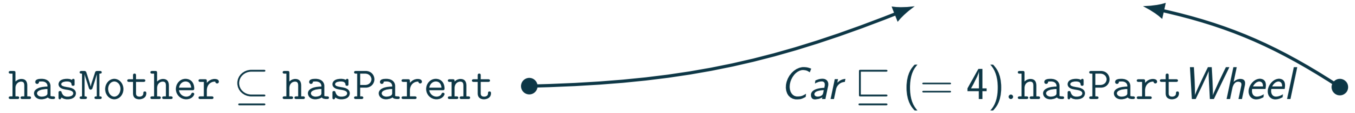

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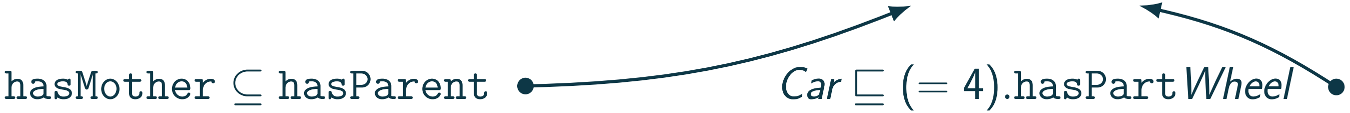

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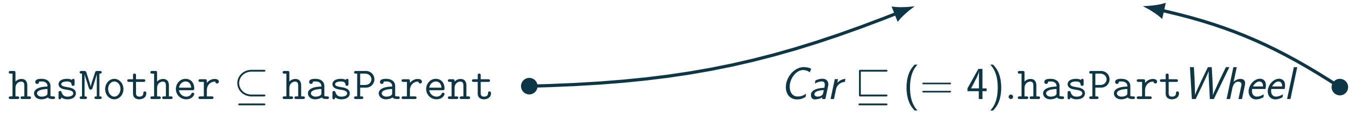

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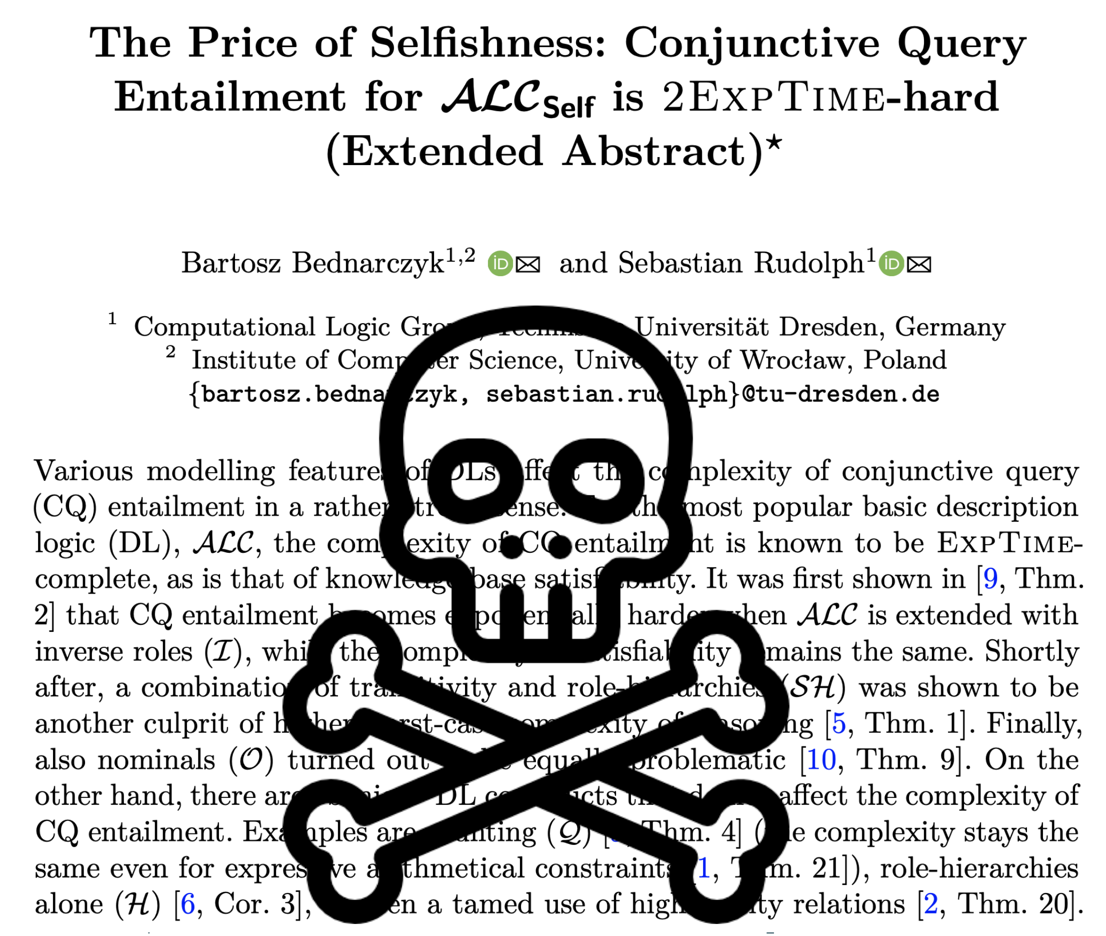

The Price of Selfishness: Conjunctive Query Entailment for $\mathcal{ALC}_{\mathsf{Self}}$ is 2ExpTime-hard (Extended Abstract)*

Bartosz Bednarczyk[1,2] and Sebastian Rudolph[1]

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Various modelling features of DLs affect the complexity of conjunctive query (CQ) entailment in a rather strong sense. For the most popular basic description logic (DL), $\mathcal{ALC}$, the complexity of CQ entailment is known to be ExpTime-complete, as is that of knowledge base satisfiability. It was first shown in [9, Thm. 2] that CQ entailment becomes exponentially harder when $\mathcal{ALC}$ is extended with inverse roles ($\mathcal{I}$), while the complexity of satisfiability remains the same. Shortly after, a combination of transitivity and role-hierarchies ($\mathcal{SH}$) was shown to be another culprit of higher worst-case complexity of reasoning [5, Thm. 1]. Finally, also nominals ($\mathcal{O}$) turned out to be equally problematic [10, Thm. 9]. On the other hand, there are various DL constructs that do not affect the complexity of CQ entailment. Examples are counting ($\mathcal{Q}$) [[illegible], Thm. 4] (the complexity stays the same even for expressive arithmetical constraints [1, Thm. 21]), role-hierarchies alone ($\mathcal{H}$) [6, Cor. 3], or even a tamed use of higher-arity relations [2, Thm. 20].

Conjunctive query entailment over $\mathcal{ALC}_{\mathsf{Self}}$ TBoxes is 2ExpTime-hard.

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†$\forall x_1\,(\mathrm{self}_{\mathrm{r}}(x_1) \to \exists x_2[\mathrm{R}(x_1,x_2) \land x_1{=}x_2]) \;\land\; \forall x_1\forall x_2\,(\mathrm{R}(x_1,x_2) \to [x_1{=}x_2 \to \mathrm{self}_{\mathrm{r}}(x_2)])$

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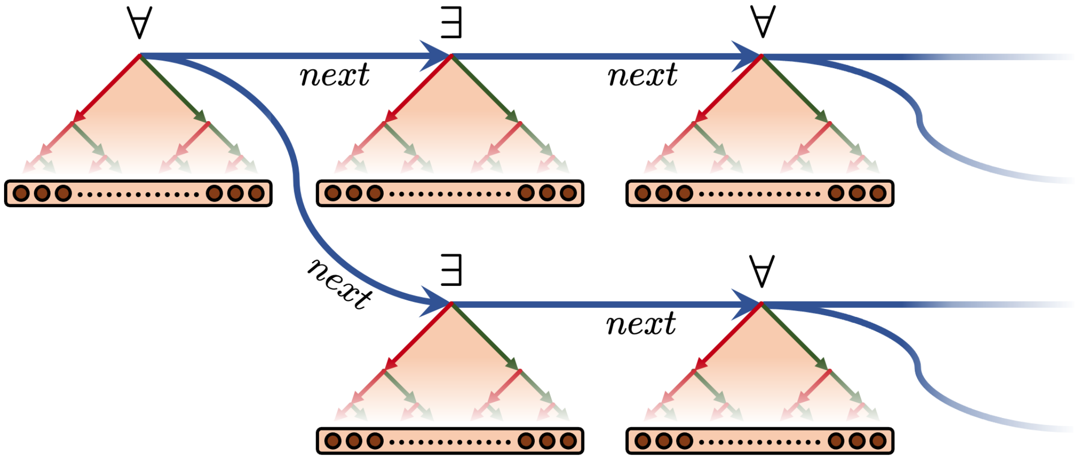

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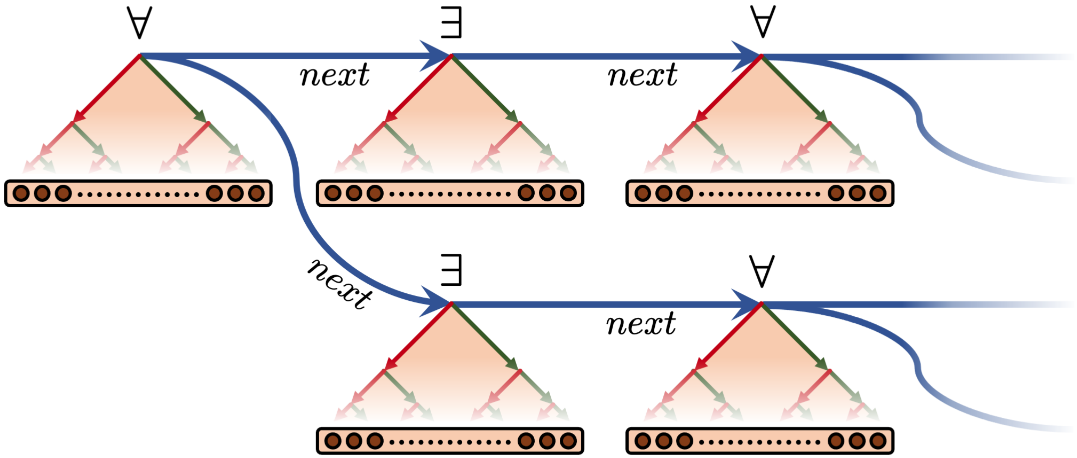

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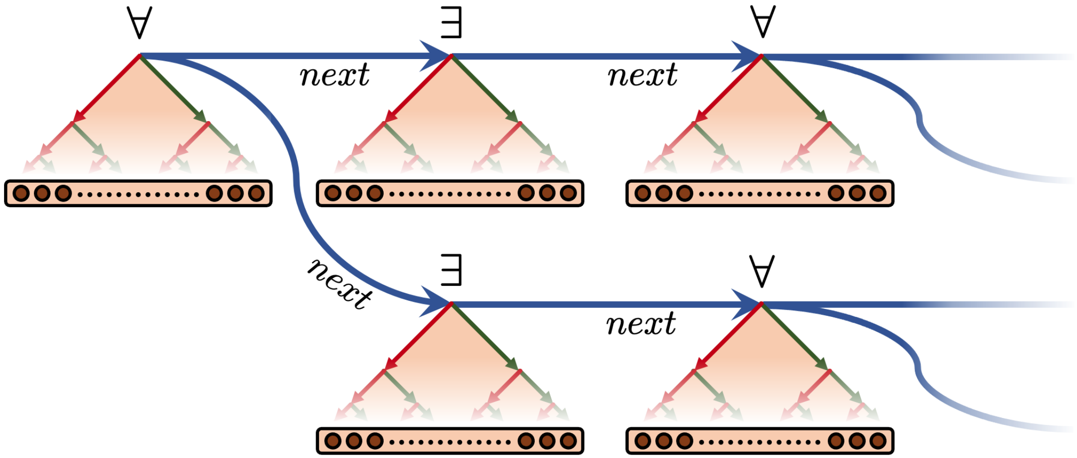

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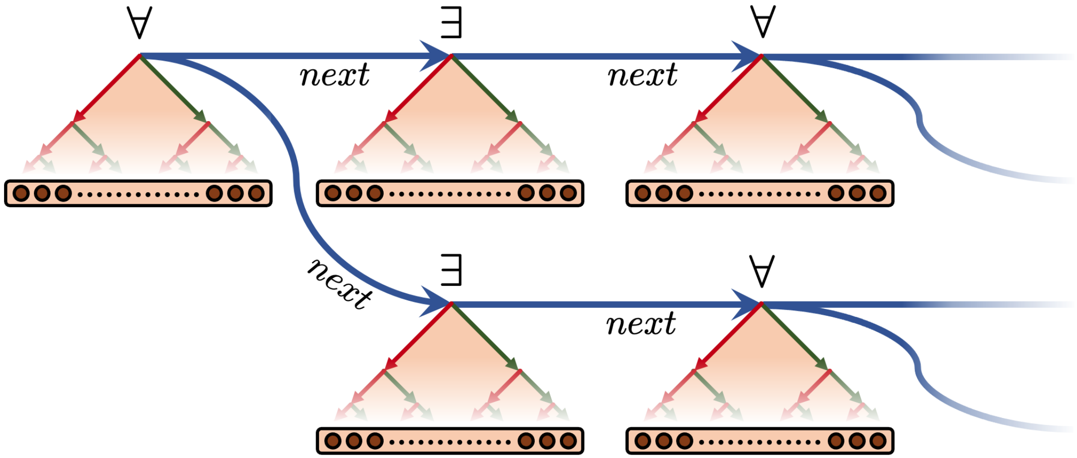

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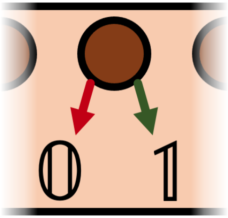

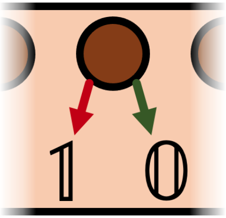

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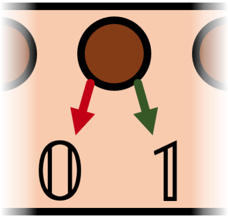

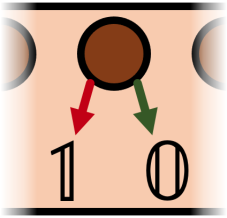

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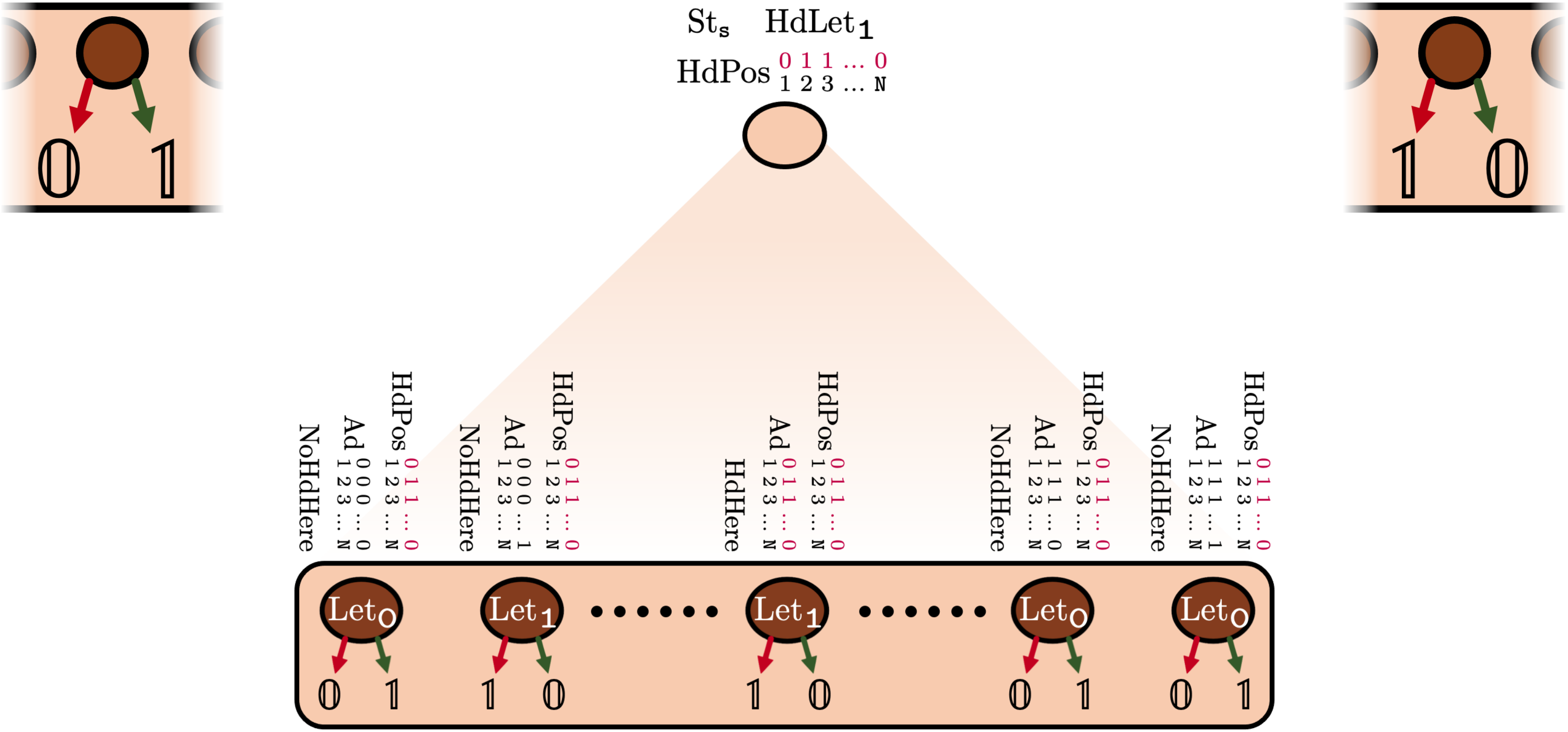

0 1
St_s HdLet_1
$\mathrm{HdPos}\ {}^{0\,1\,1\,\ldots\,0}_{1\,2\,3\,\ldots\,N}$
1 0
NoHdHere
$\mathrm{Ad}\ {}^{0\,0\,0\,\ldots\,0}_{1\,2\,3\,\ldots\,N}$
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Let_0 Let_1 …… Let_1 …… Let_0 Let_0
0 1 1 0 1 0 0 1 0 1

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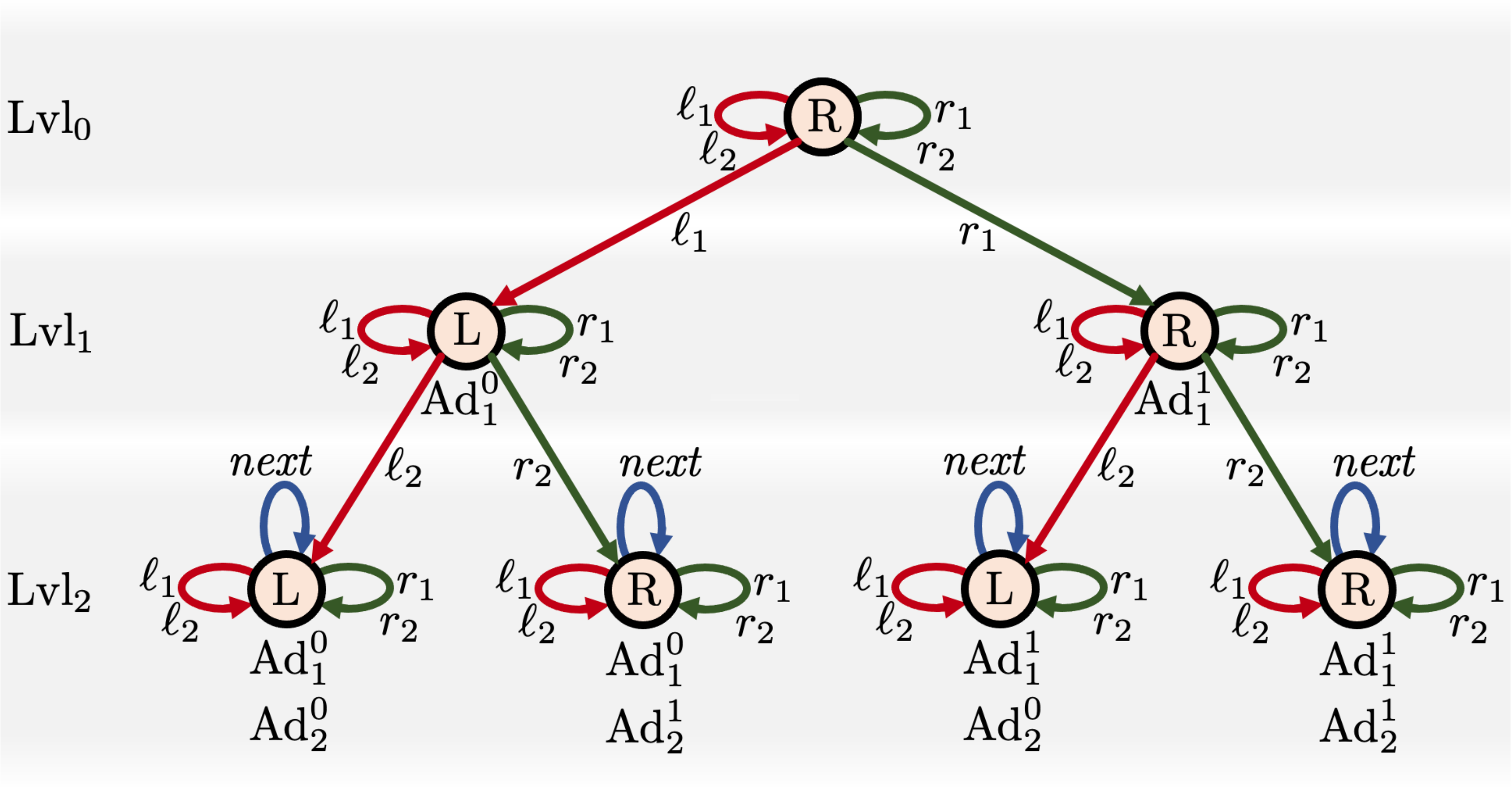

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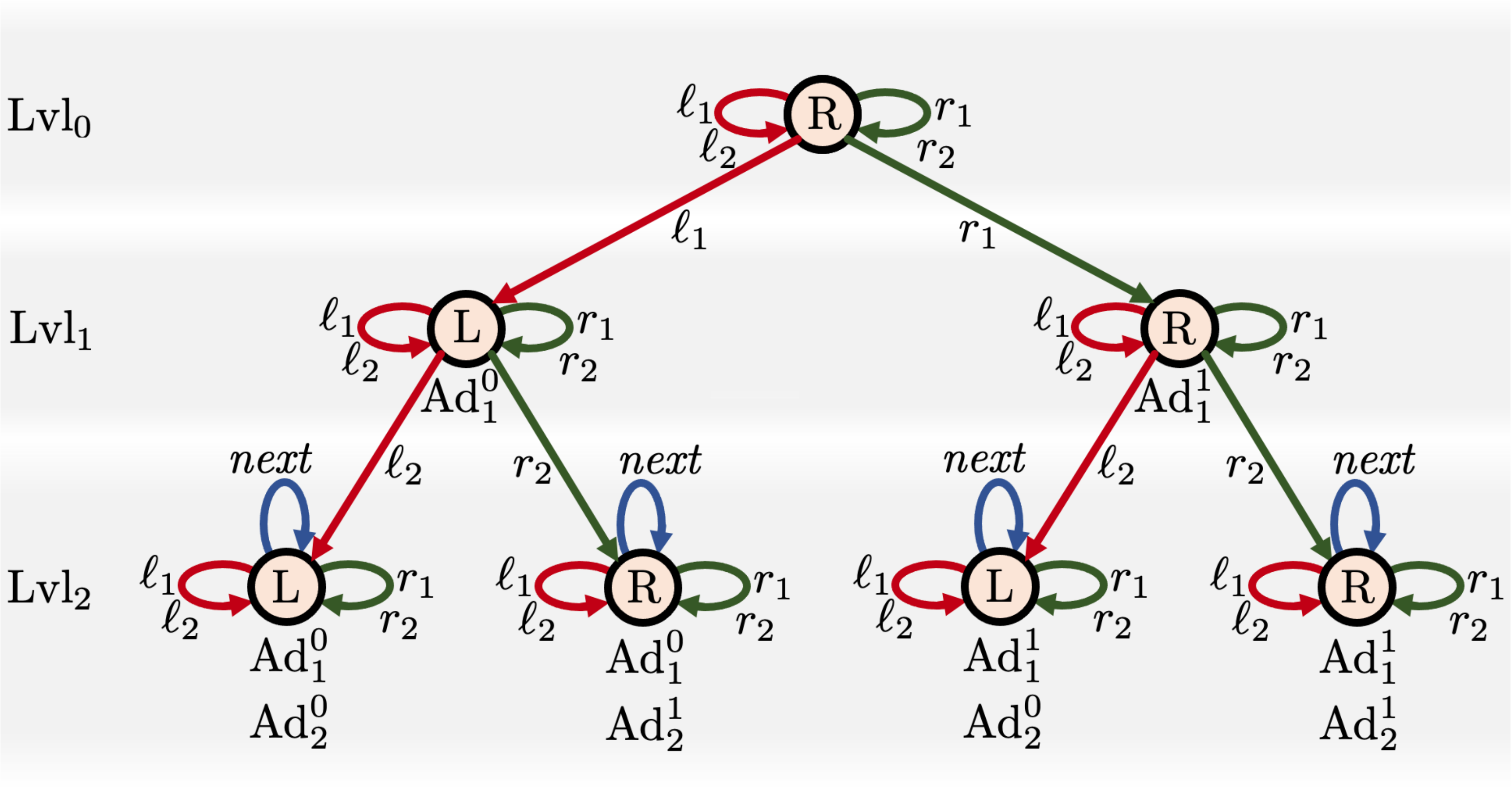
Goal: Design a CQ $q(x, y)$ such that x matches the root and y matches any of the leaves.



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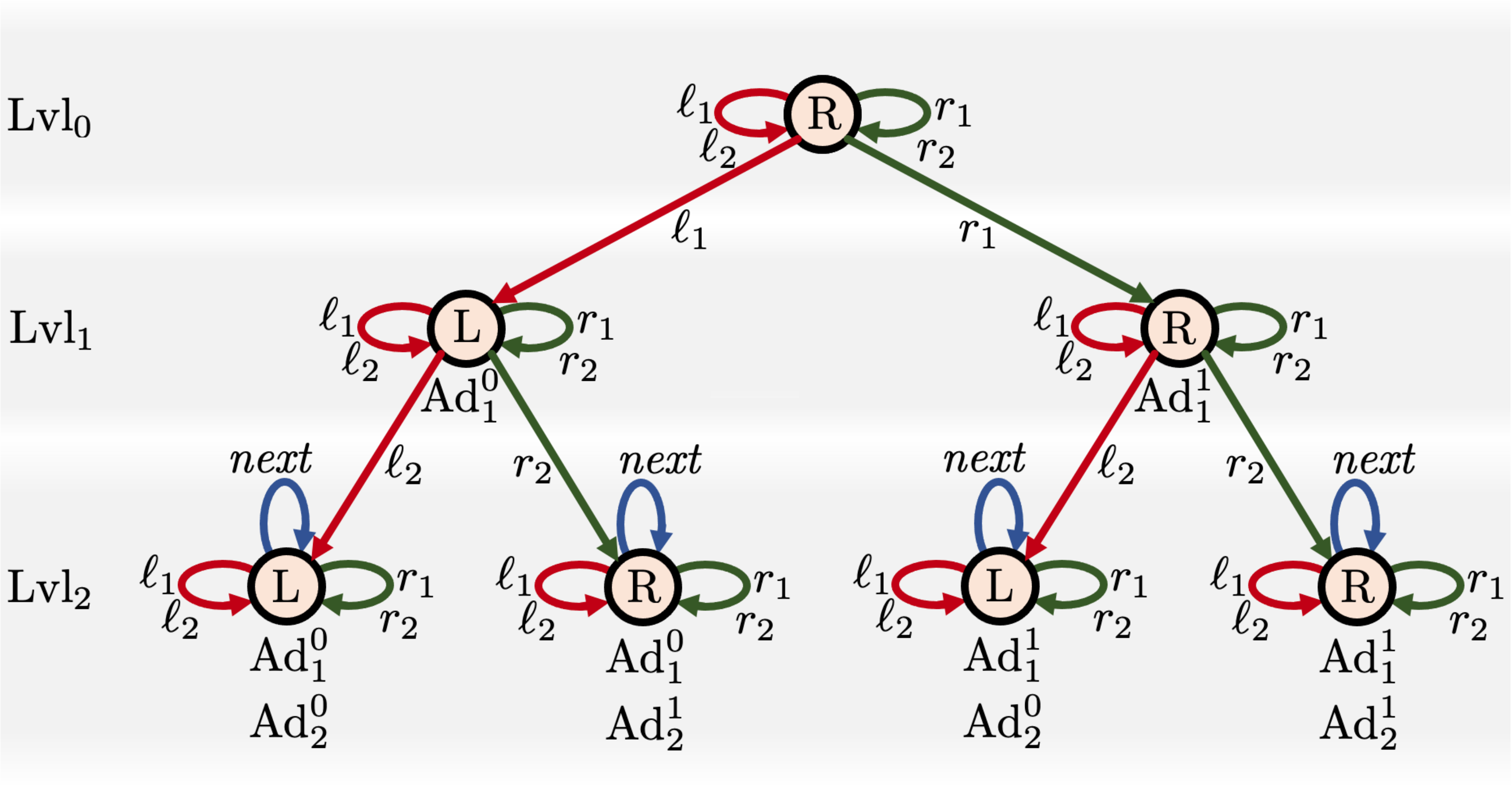

$$\exists x_1 \exists x_2 \exists x_3 \; Lvl_0(x) \wedge \ell_1(x, x_1) \wedge \mathrm{r}_1(x_1, x_2) \wedge \ell_2(x_2, x_3) \wedge \mathrm{r}_2(x_3, y) \wedge Lvl_2(y)$$

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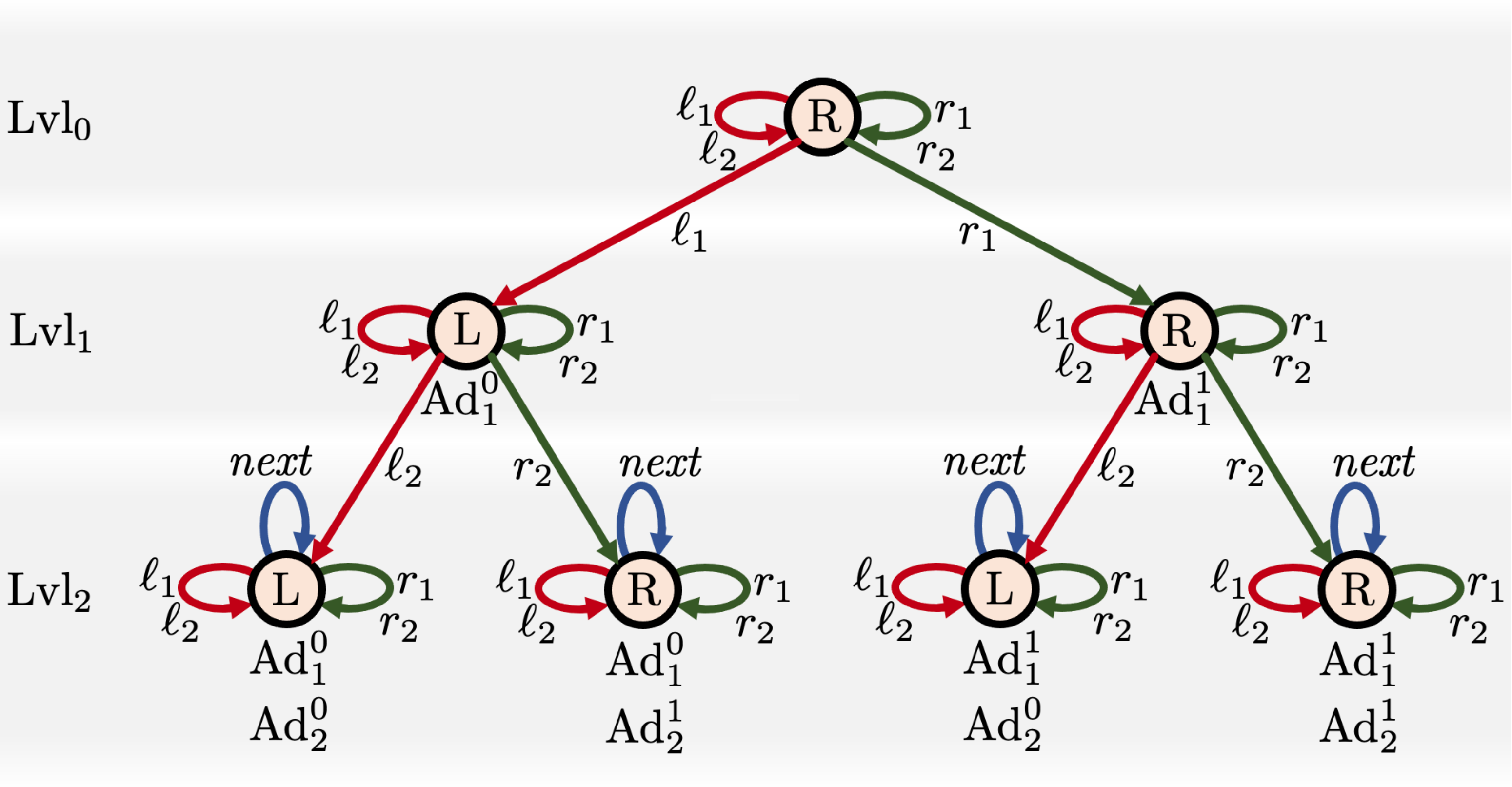

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For brevity we write: $(Lvl_0?; \ell_1; \mathrm{r}_1; \ell_2; \mathrm{r}_2; Lvl_2?)(x, y)$.

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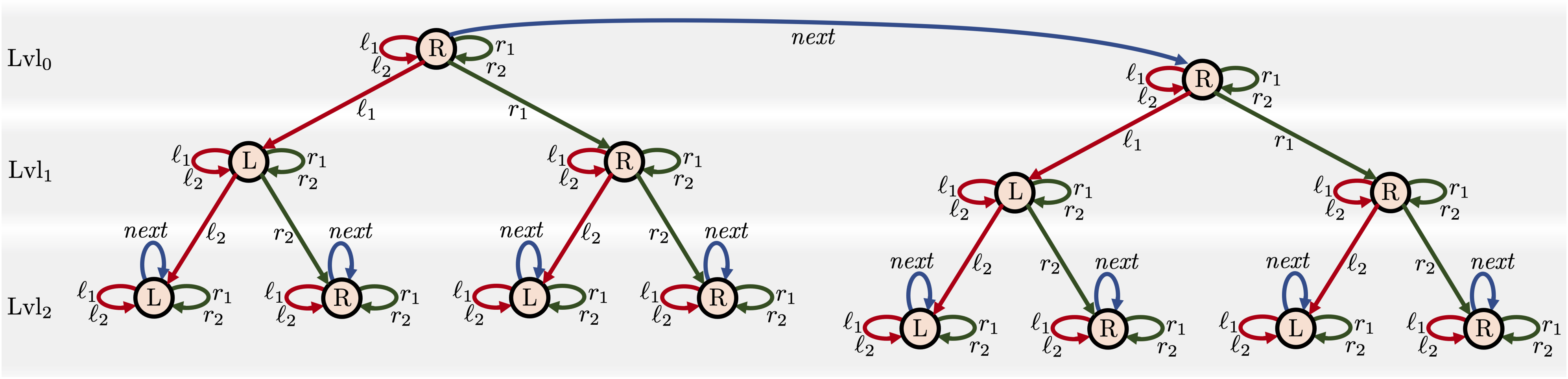

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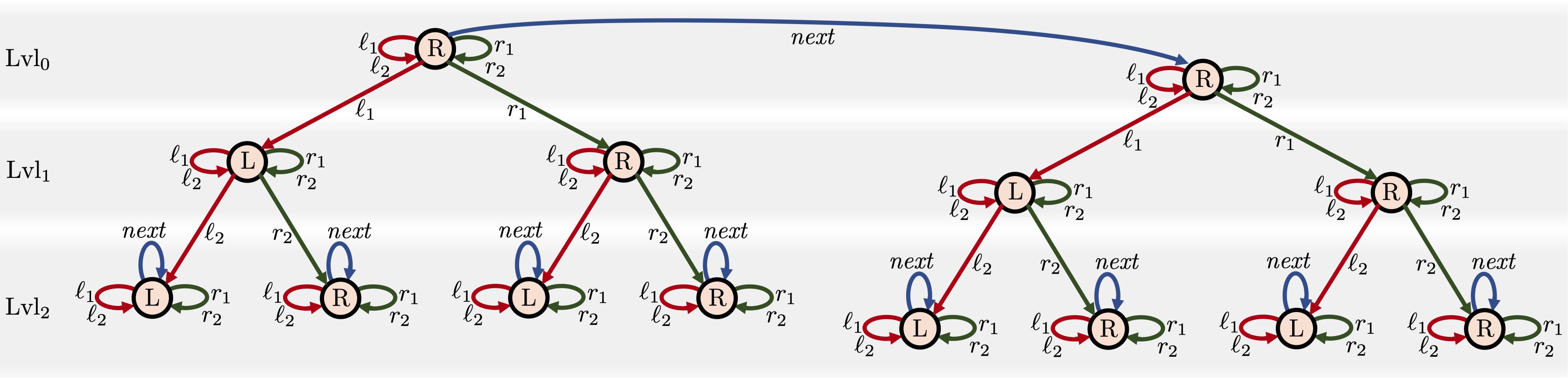

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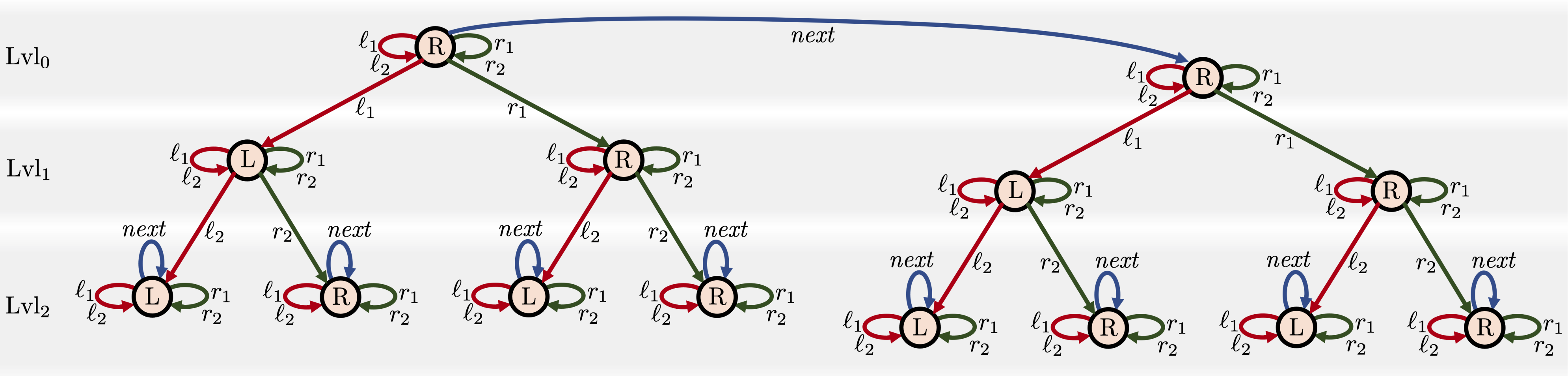

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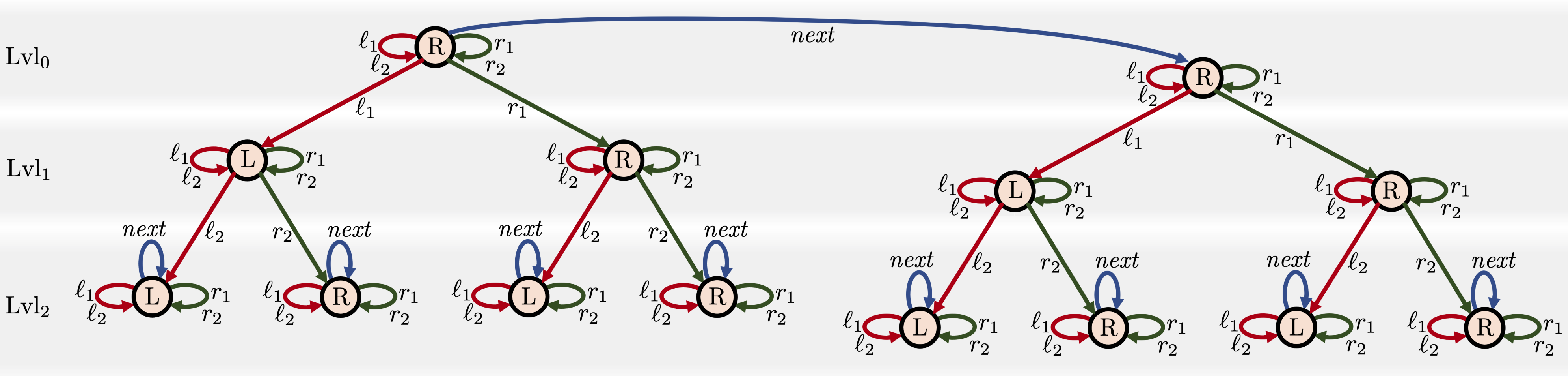

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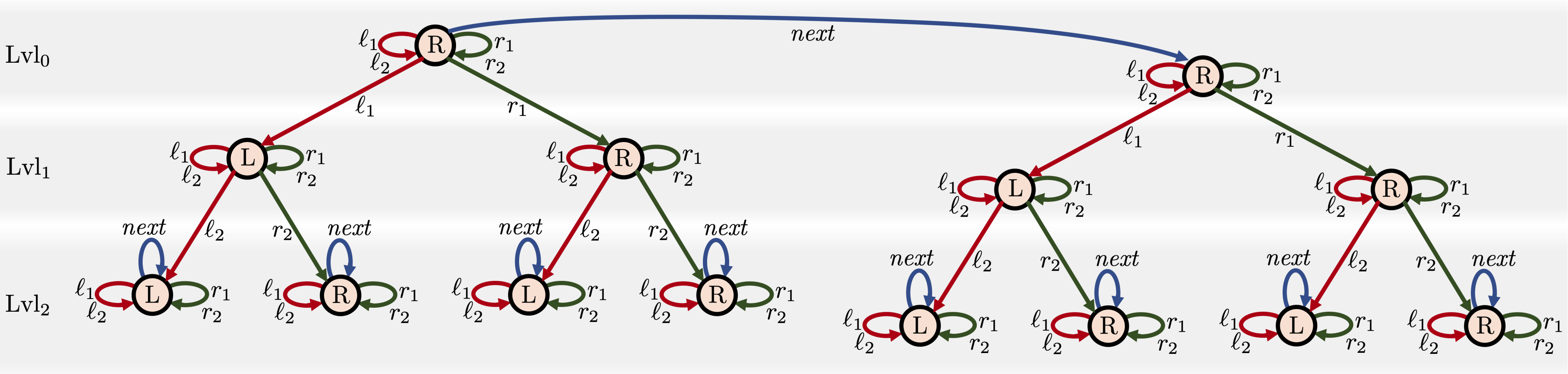

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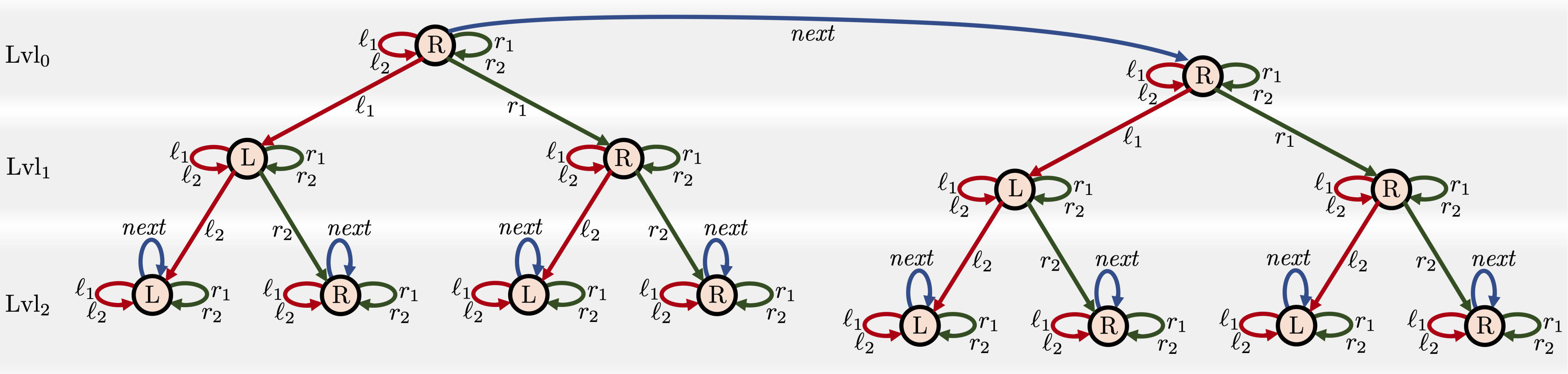

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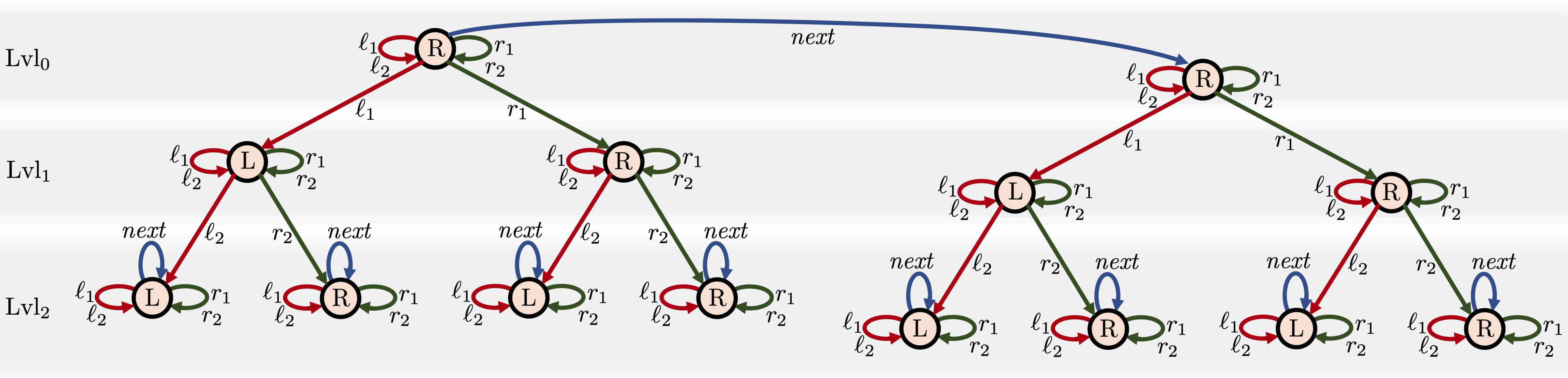

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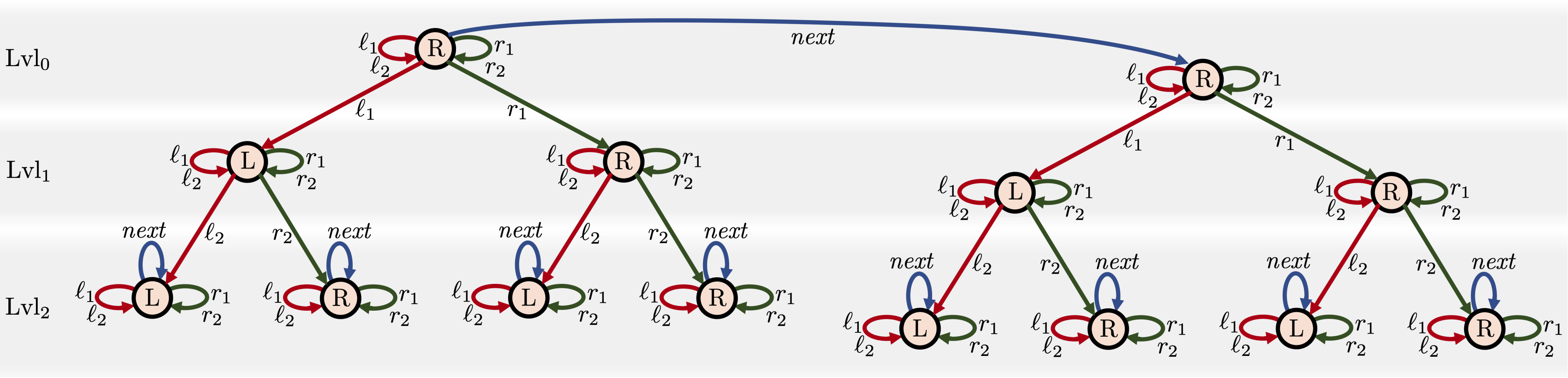

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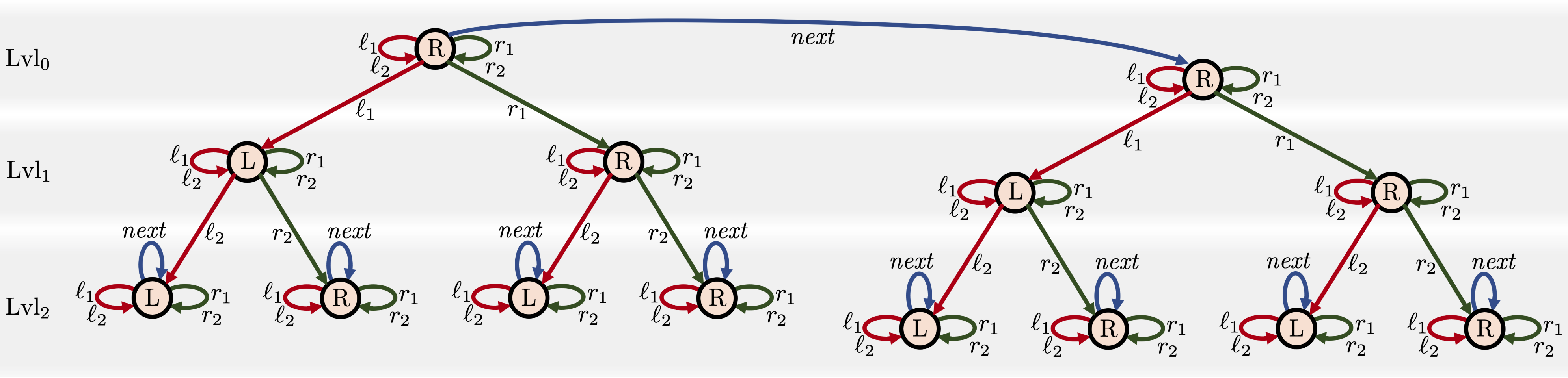
The end: Thanks for your attention!

Biggest challenge: Design a CQ $q(x, y)$ that matches leaves x, y with equal addresses.



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