A tamed higher-arity extension of $\mathcal{ALC}$

Forward Guarded Fragment

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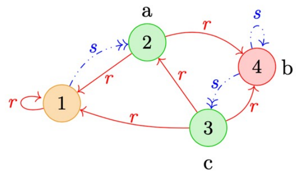

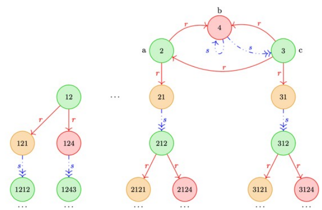

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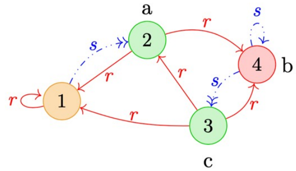

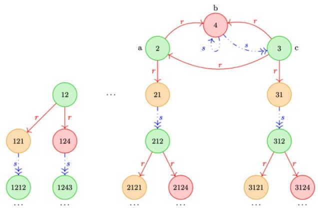

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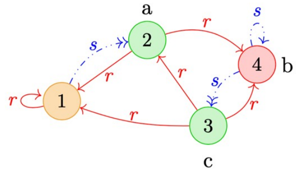

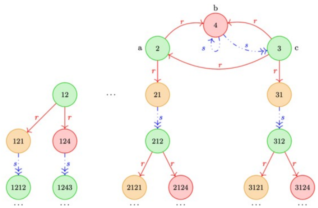

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Yes! $\mathcal{FGF}$ [B. JELIA'21, This talk!]

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Theorem (Bárány et al. 2013)

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Lemma (B. 2021)

$\mathcal{FF}$ is reducible to $\mathcal{FL}$ in polynomial time.

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In $\mathcal{GF}$:

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LTL corresponds to $\mathcal{FO}[\leq]$ over words, LTL[$\mathbf{XF}, \mathbf{XP}$] corresponds to $\mathcal{FO}^2[\leq]$

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Theorem (TFAE for a formal language $\mathcal{L} \subseteq \Sigma^*$)

(a) $\mathcal{L}$ is definable in $\mathcal{FGF}[\leq]$, (b) is def. in LTL[$\mathbf{XF}$],

(c) is rec. by partially-ordered 1way DFA, (d) $\mathrm{M}(\mathcal{L})$ belongs to the variety $\mathbf{R}$

(e) $\mathcal{L}$ is a fin disj. union $A_0^* a_1 A_1^* \ldots a_k A_k^*$ with $a_i \in \Sigma$, $A_i \subseteq \Sigma$ and $a_i \notin A_{i-1}$.

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Corollary

Data complexity of KB SAT is NP-compl and coNP-compl for querying.

$\mathcal{FGF}$ has FMP and is finitely-controllable.

Main ingredients for SAT: Part I

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Definition (Forward type)

A (Σ, n)-*forward type* is a conjunction of atoms with n free-variables $\vec{x}_{1 \ldots n}$, which for every relational symbol $\mathrm{R} \in \Sigma$ of arity $\ell = \mathrm{ar}(\mathrm{R}) \leq n$ and every index $1 \leq i \leq n+1-\ell$ contains either $\mathrm{R}(\vec{x}_{i \ldots i+\ell-1})$ or $\neg\mathrm{R}(\vec{x}_{i \ldots i+\ell-1})$.

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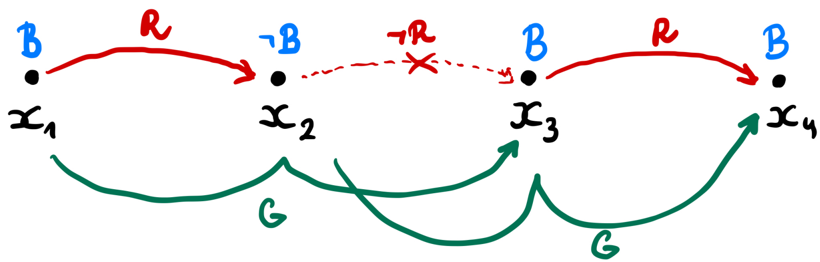
$(\{R, G, B\}, 4)$-forward tp



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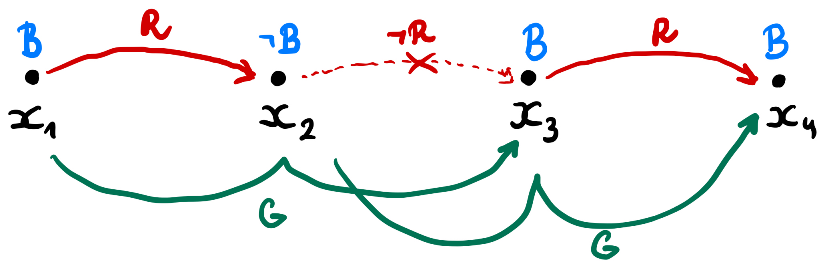

Lemma

The number of different (Σ, n)-types is $\leq 2^{|\Sigma| \cdot n^2}$.

The number of conjuncts in each (Σ, n)-type is $\leq |\Sigma| \cdot n$

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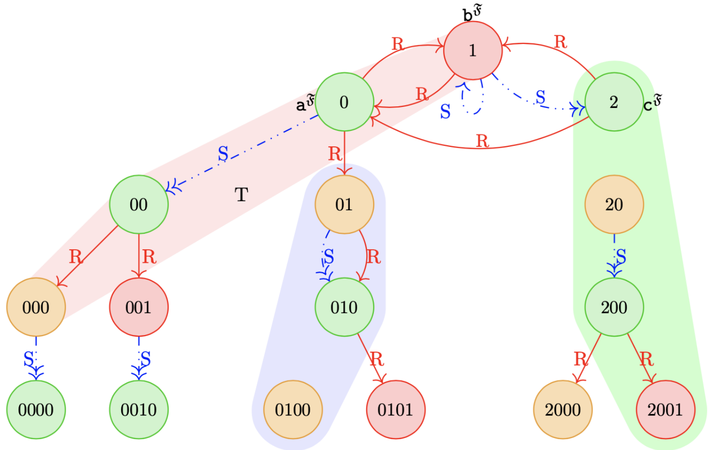

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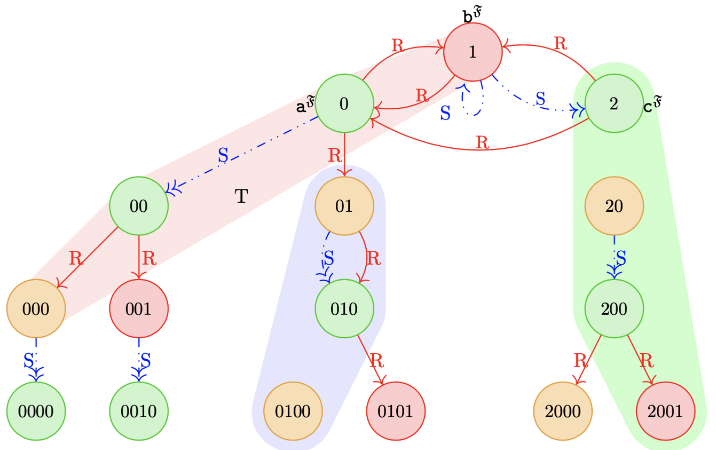

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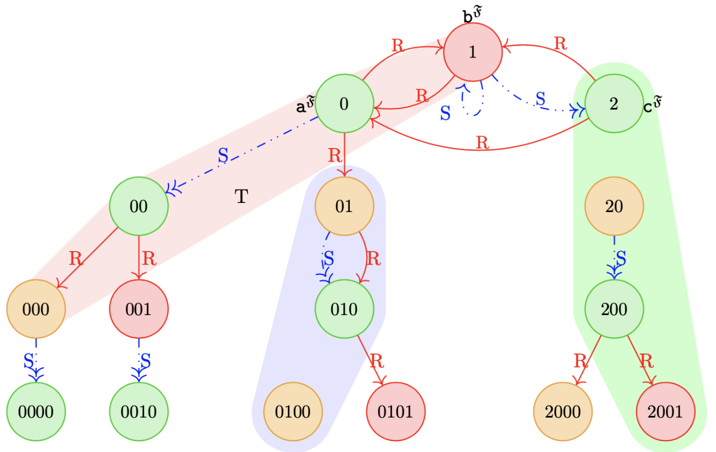

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Proof

via suitable notion of HAF-unravelling, similar to [BBR, ECAI'20]

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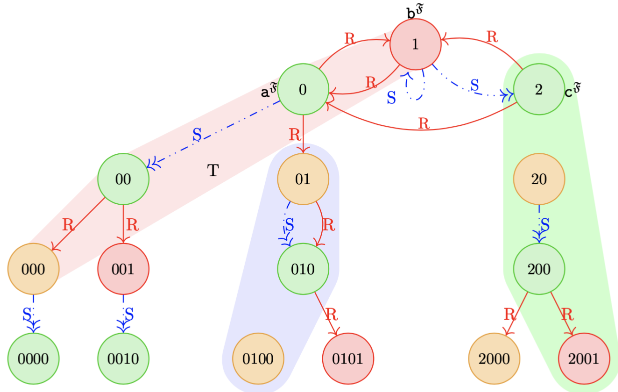

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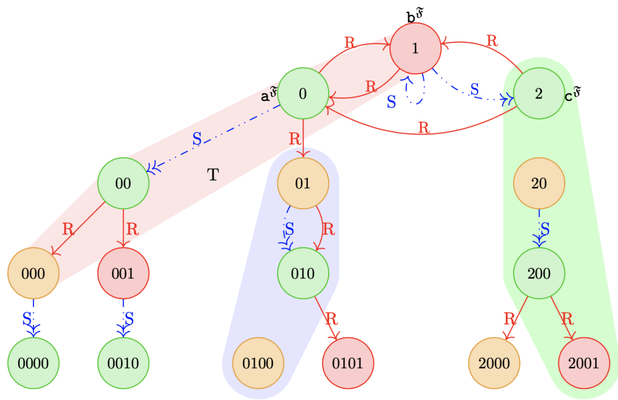

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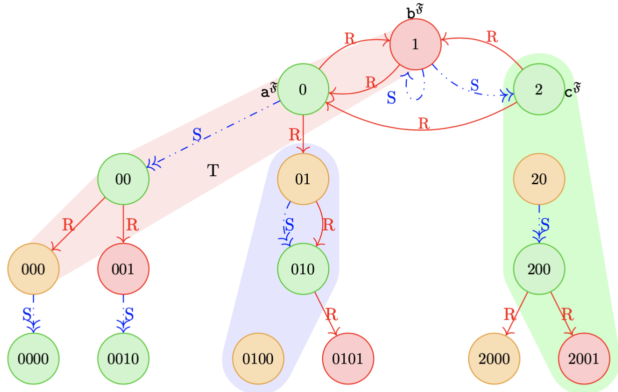

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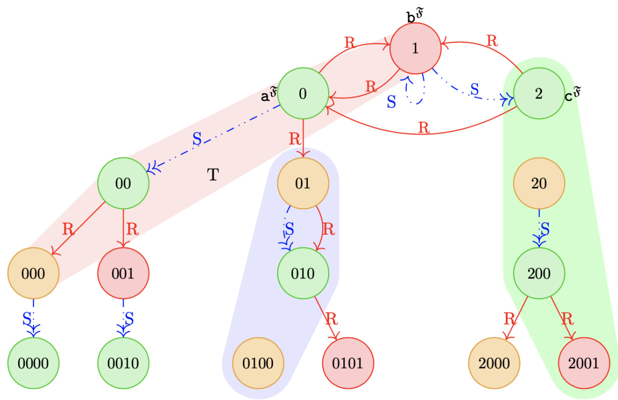

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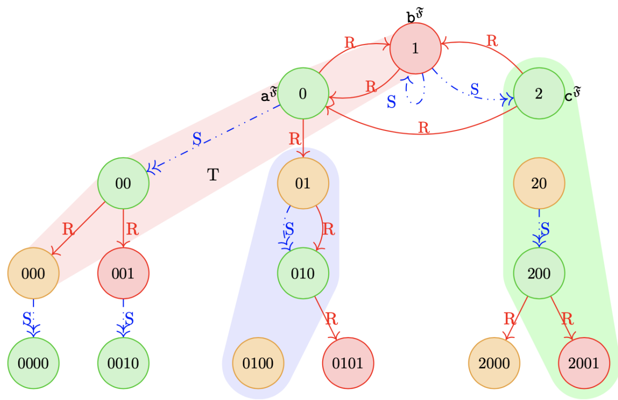

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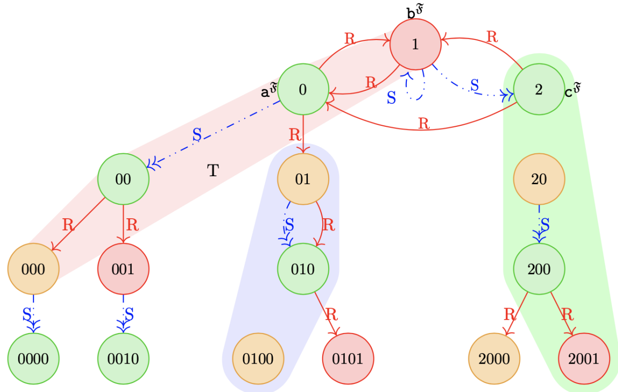

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Quite technical generalisation of the rolling-up technique of transforming tree-shaped matches into concepts.

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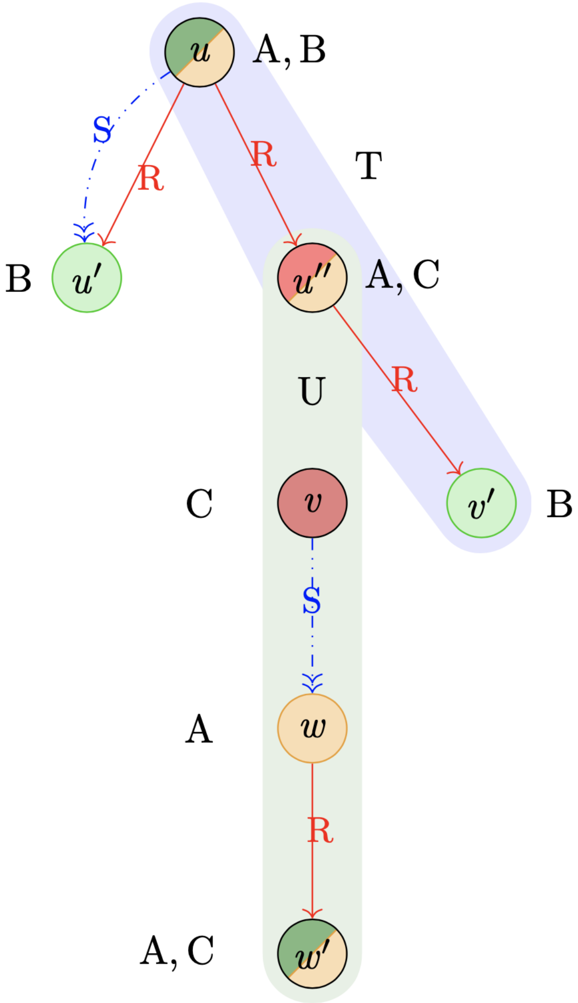

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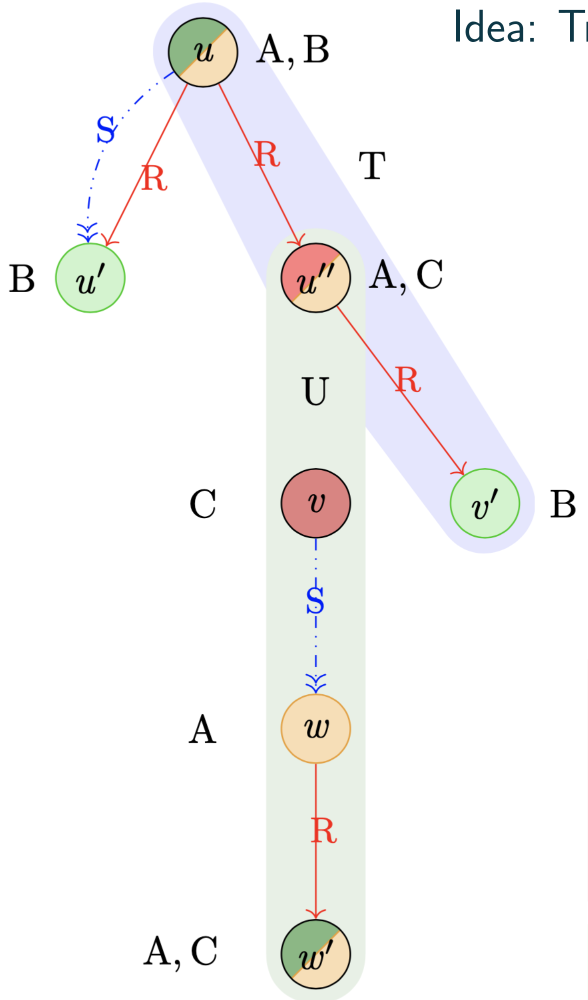

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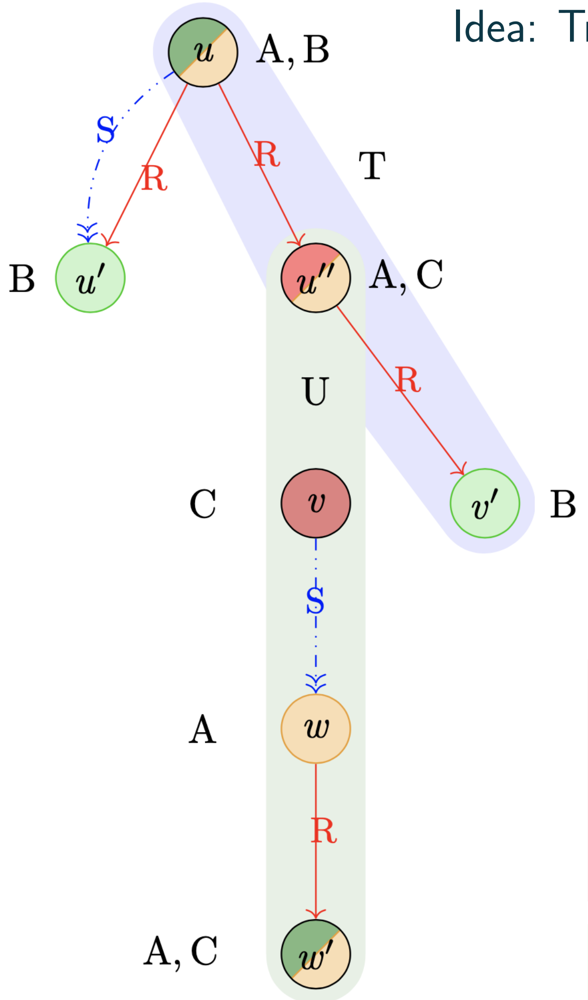

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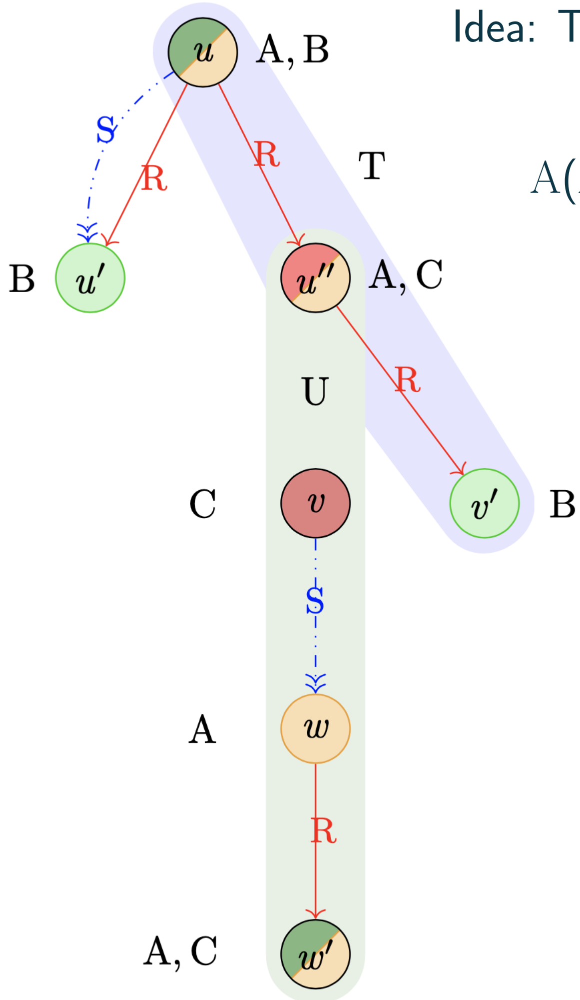

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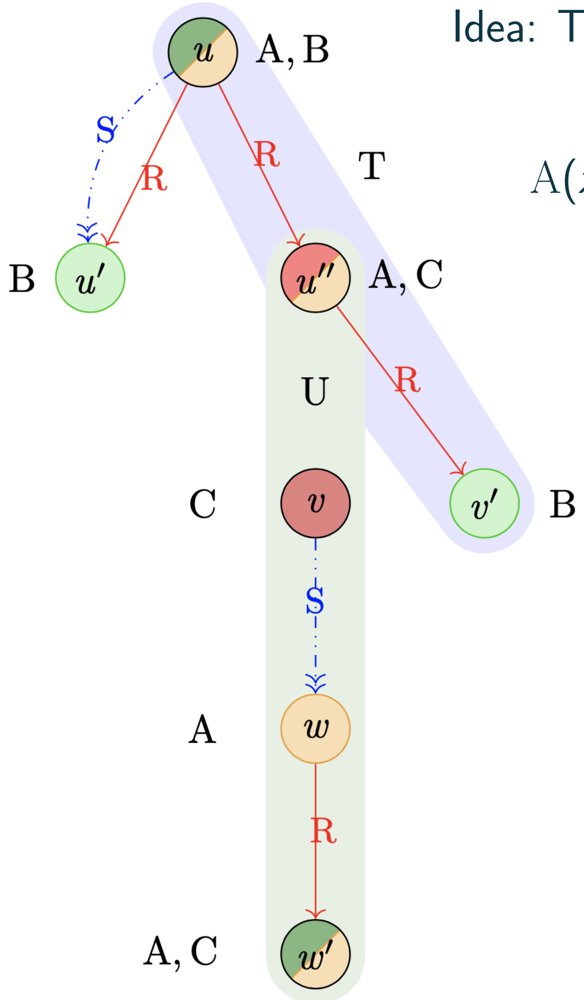

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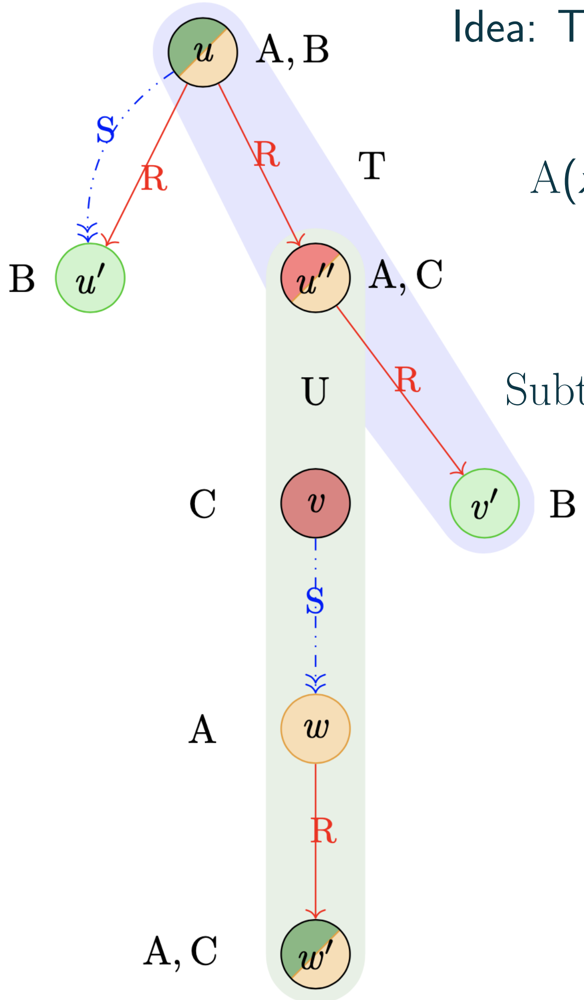

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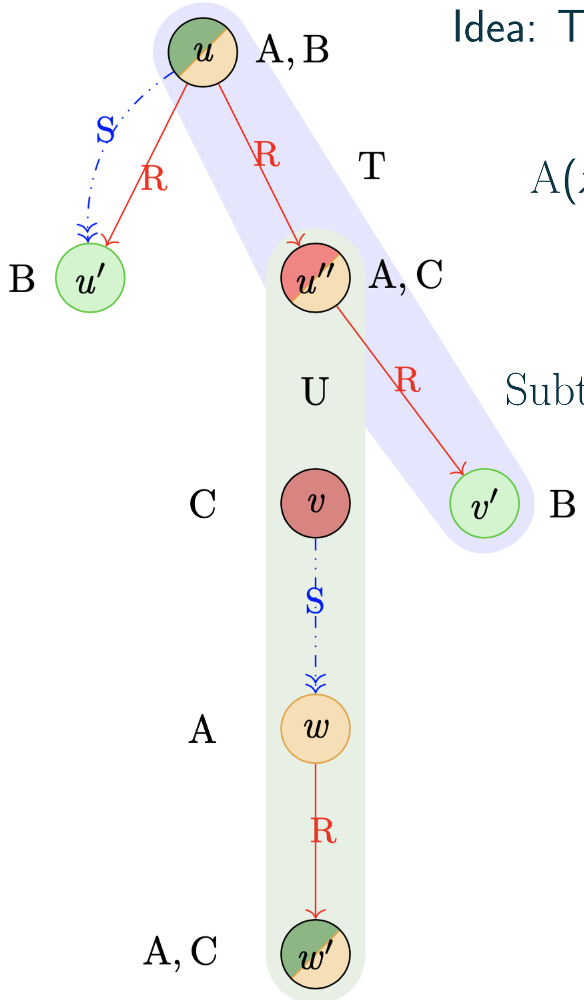

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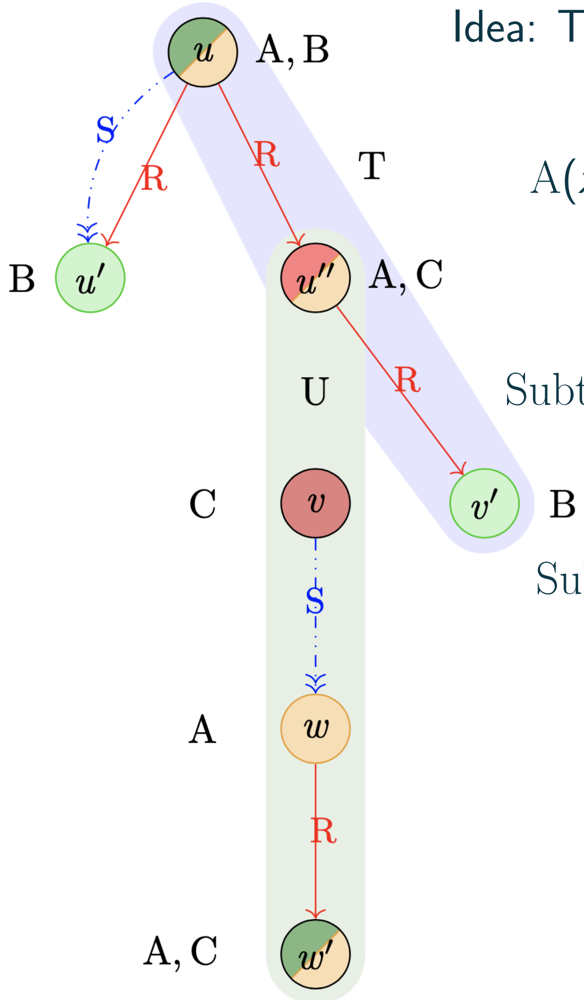

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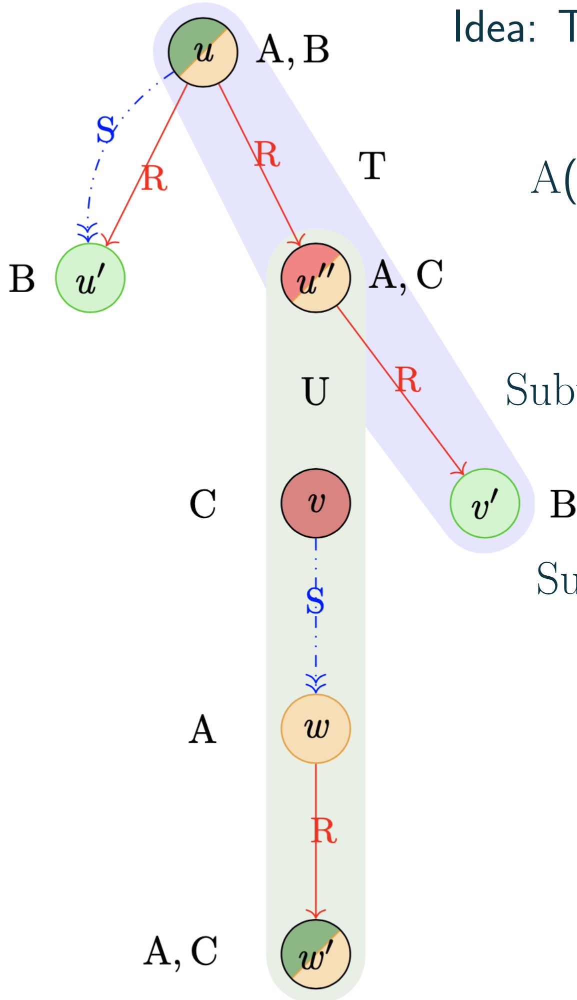

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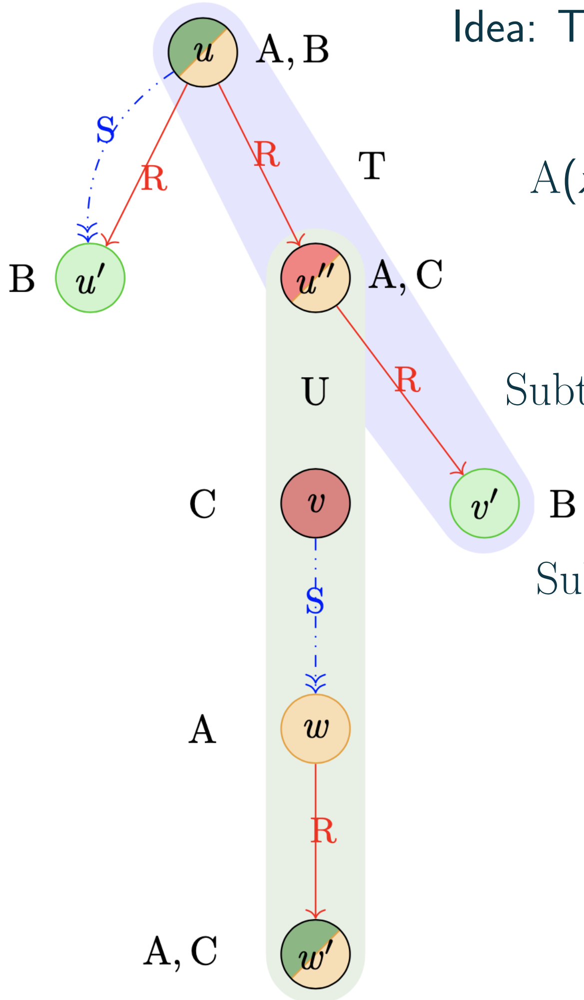

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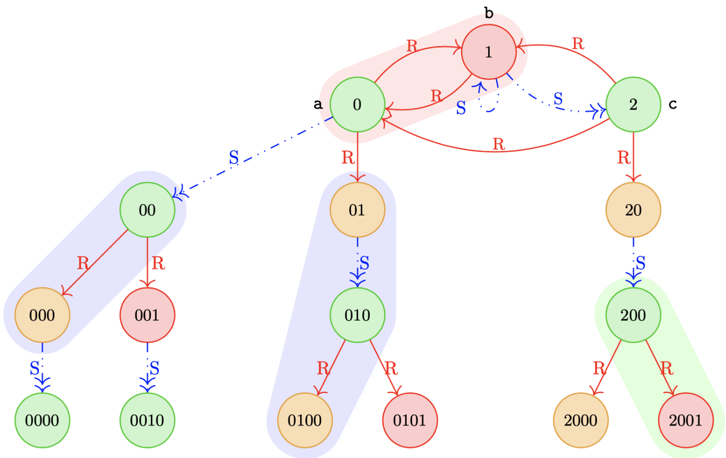

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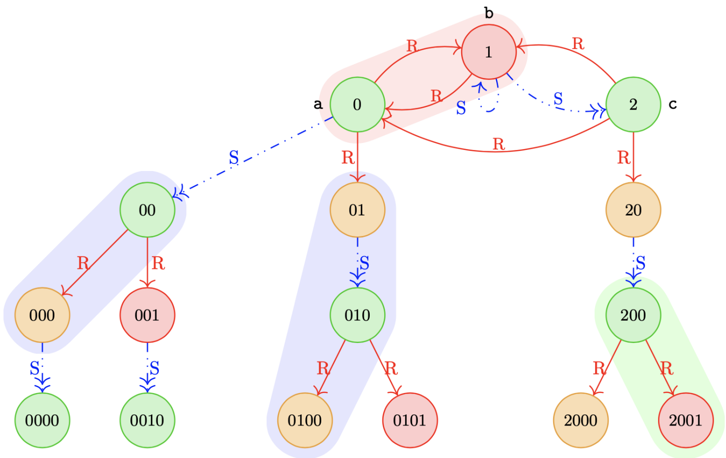

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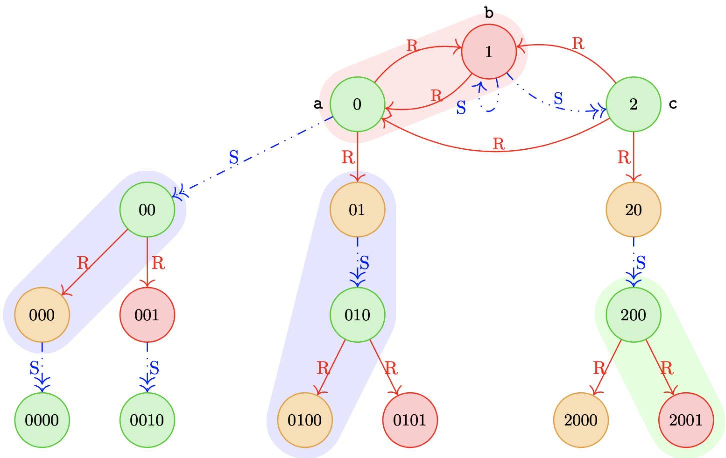

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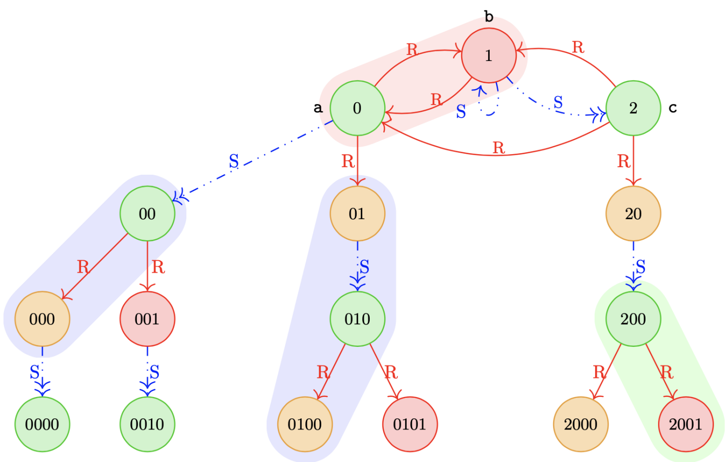

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To construct a spoiler we must know how to "describe" Π in $\mathcal{FGF}$, in particular cases (a), (b) and (c).

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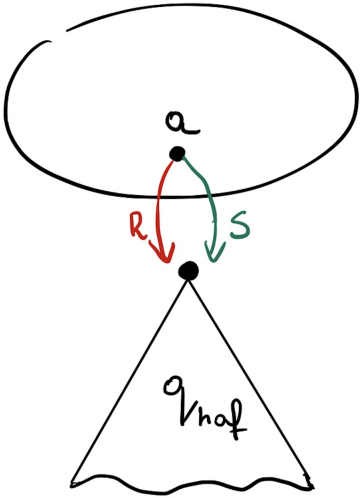

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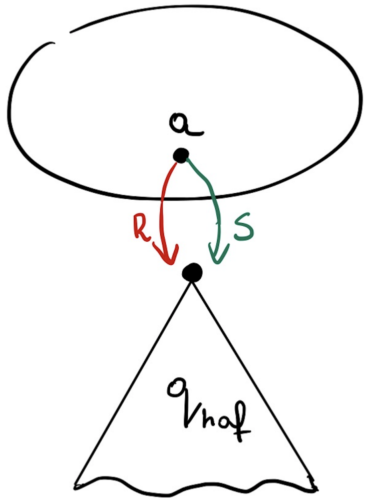

Simply insert

$$(\exists x_2 \, \mathrm{R}(x_1, x_2) \wedge \mathrm{R}(x_1, x_2) \wedge \mathrm{Match}_{q_{\mathsf{haf}}}(x_2)) \, (\mathtt{a})$$

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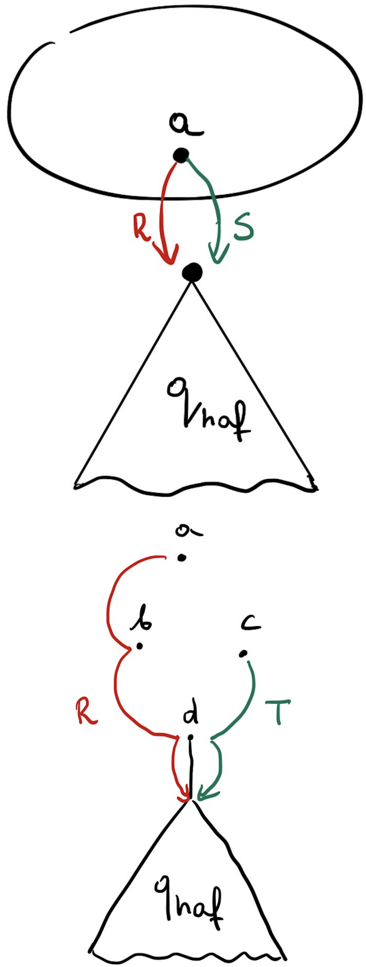

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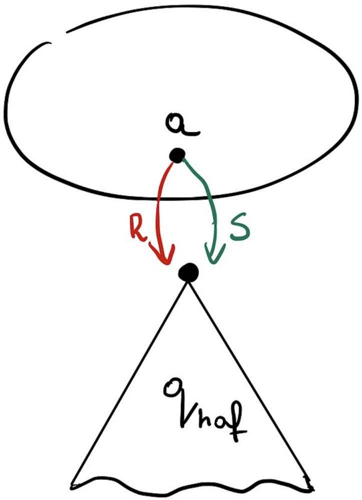

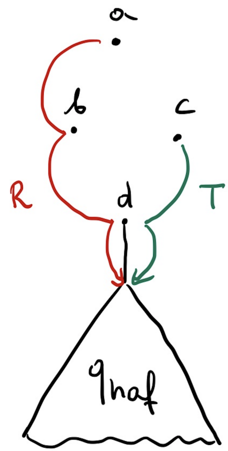

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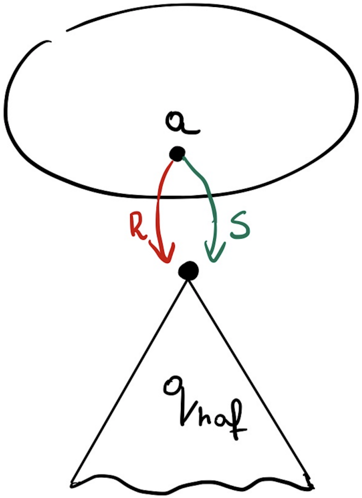

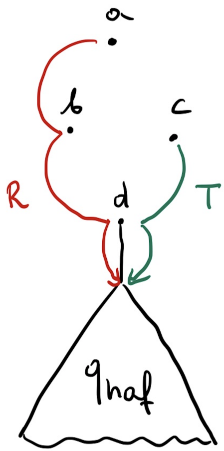

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Repair idea: introduce a bit more constants to $\mathcal{FGF}$ but not too much.

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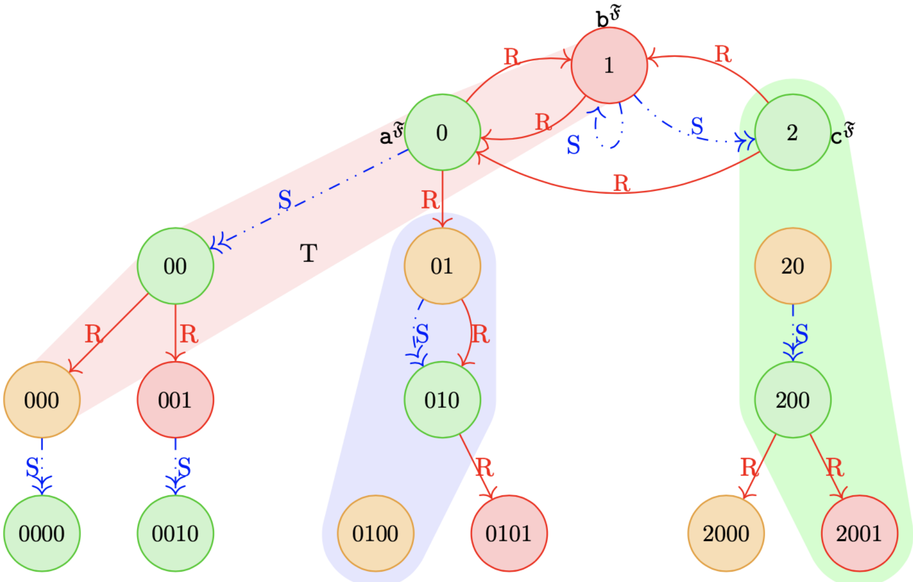

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Thanks for attention!