Towards a Model Theory of Ordered Logics
Expressivity and Interpolation

August 23rd 2022, Vienna, MFCS 2022

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TU Dresden & University of Wrocław

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Tampere University

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Our motivations

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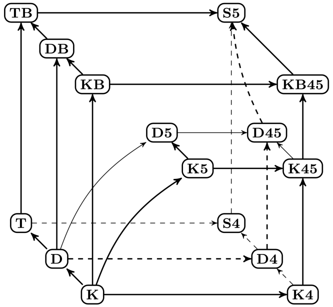

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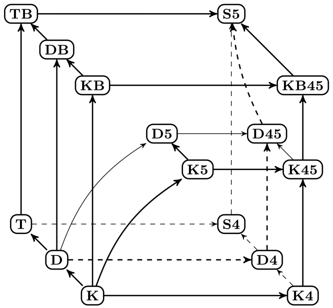

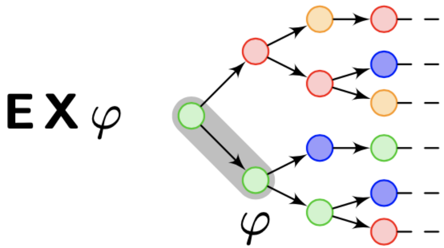

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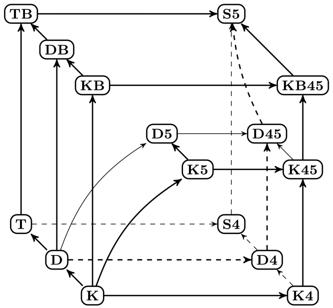

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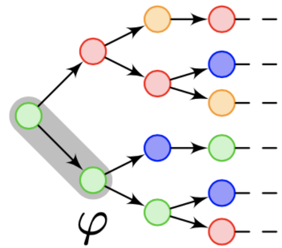

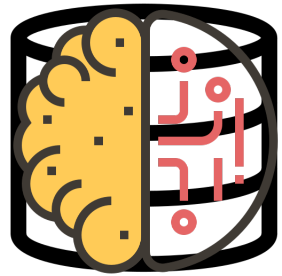

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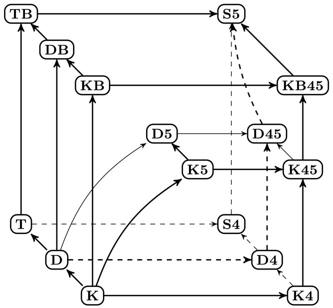

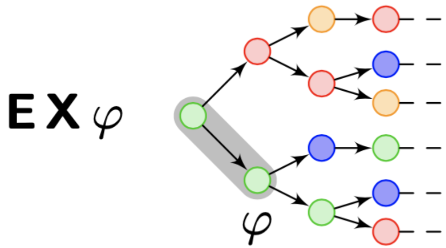

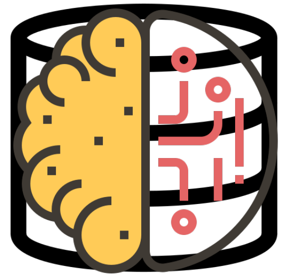

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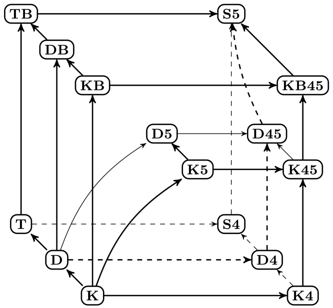

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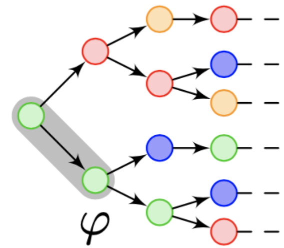

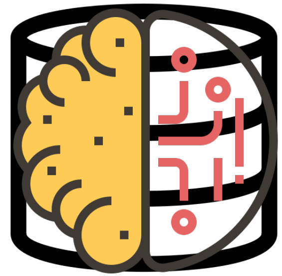

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  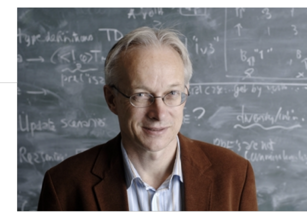

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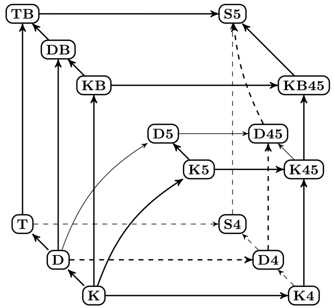

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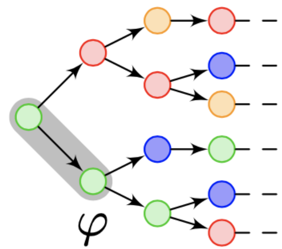

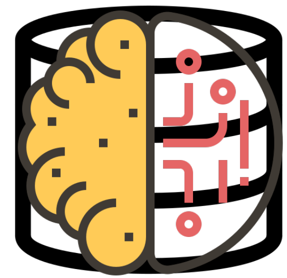

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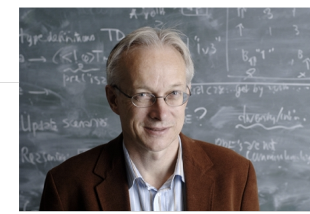

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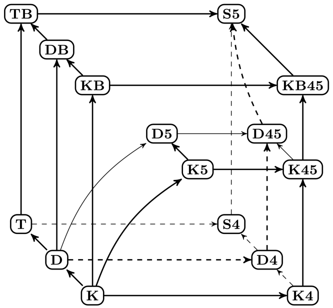

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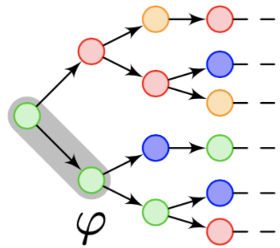

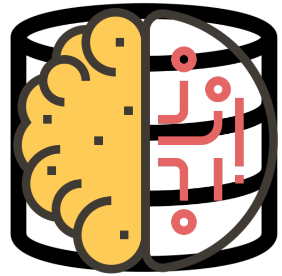

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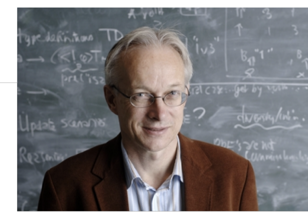

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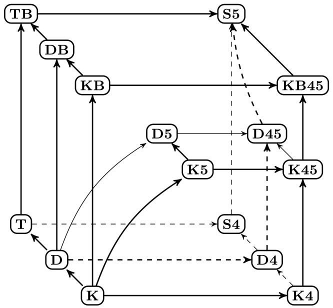

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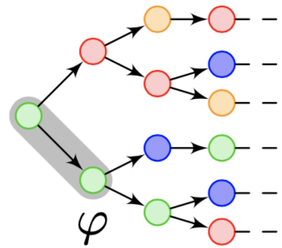

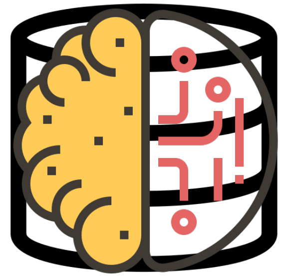

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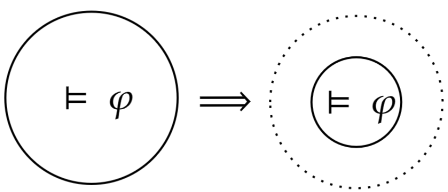
Łoś-Tarski Preservation Thm. (ŁTPT)



$\Longleftrightarrow \quad \varphi \equiv \forall^* \varphi'$

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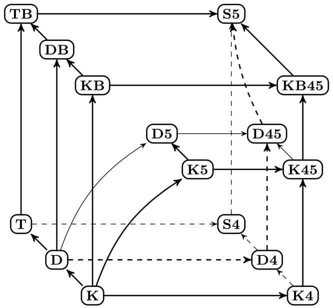

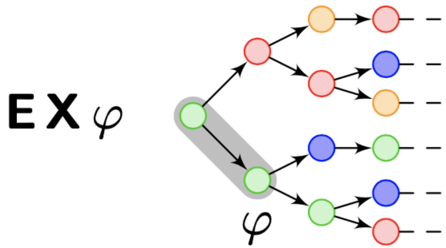

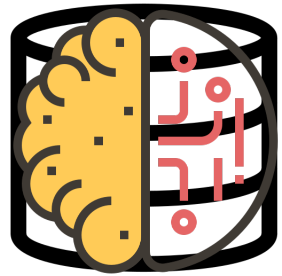

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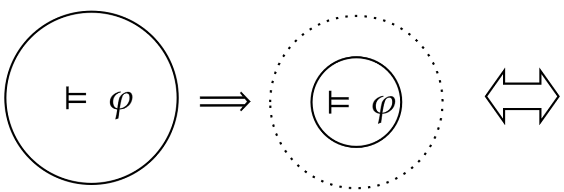
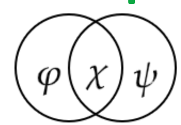


$\models \varphi \Longrightarrow \models \varphi \iff \varphi \equiv \forall^* \varphi'$

Craig Interpolation Property (CIP)



$sig(\chi) \subseteq sig(\varphi) \cap sig(\psi)$

$\varphi \models \psi \Longrightarrow \exists \chi \;\; \varphi \models \chi \models \psi$

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- $\exists \vec{y}\ \alpha(\vec{x}, \vec{y}) \wedge \varphi(\vec{x}, \vec{y}), \forall \vec{y}\ \alpha(\vec{x}, \vec{y}) \rightarrow \varphi(\vec{x}, \vec{y})$ – guard must cover free variables of φ.

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$$\exists x\ artst(x) \wedge \forall y\ \ (adm(x, y) \rightarrow bkpr(y))$$

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Example 2. Every artist envies every beekeeper he admires

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$$\forall x_1(stud(x_1) \rightarrow \neg\forall x_2(prof(x_2) \rightarrow admires(x_1, x_2)))$$

Example 2. No lecturer introduces any professor to every student

$$\forall x_1(lect(x_1) \rightarrow \neg\exists x_2(prof(x_2) \wedge \forall x_3(stud(x_3) \rightarrow intro(x_1, x_2, x_3))))$$

Coexample 1. $\forall x_1 r(x_1, x_1)$

Coexample 2. $\forall x_1 \forall x_2 r(x_1, x_2) \rightarrow s(x_2, x_1)$

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SAT

PSpace/Tower-complete

Two nice logics: $\mathcal{GF}$ [Andreka et al. 1998] and ordered logics [Herzig, Quine, B.]

- The ordered fragments $\mathsf{L}_{\mathrm{pre}}, \mathsf{L}_{\mathrm{suf}}, \mathsf{L}_{\mathrm{inf}}$ of $\mathcal{FO}$ are obtained by keeping the variables ordered.
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SAT	FMP	CIP
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SAT	FMP	CIP	ŁTPT
PSPACE/TOWER-complete			

On the infamous work of Purdy

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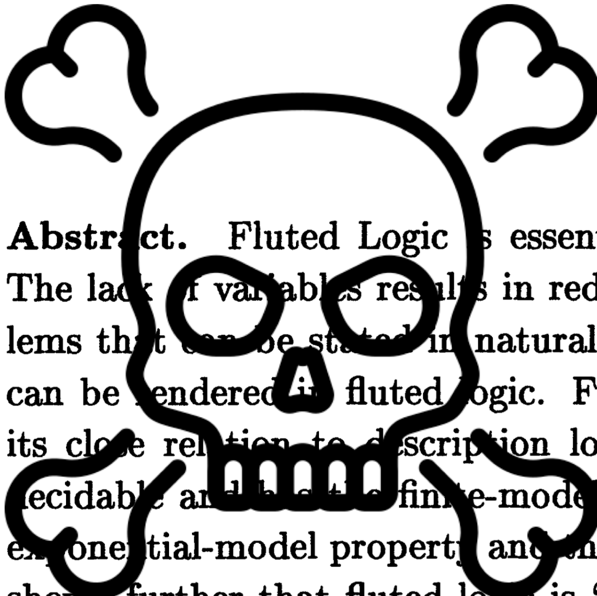
William C. Purdy

Complexity and Nicety of Fluted Logic



Abstract. Fluted Logic is essentially first-order predicate logic deprived of variables. The lack of variables results in reduced expressiveness. Nevertheless, many logical problems that can be stated in natural language, such as the famous Schubert's Steamroller, can be rendered in fluted logic. Further evidence of the expressiveness of fluted logic is its close relation to description logics. Already it has been shown that fluted logic is decidable and has the finite-model property. This paper shows that fluted logic has the exponential-model property and that deciding satisfiability is NEXPTIME-complete. It is shown further that fluted logic is 'nice', that is, it shares with first-order predicate logic the interpolation property and model preservation properties.

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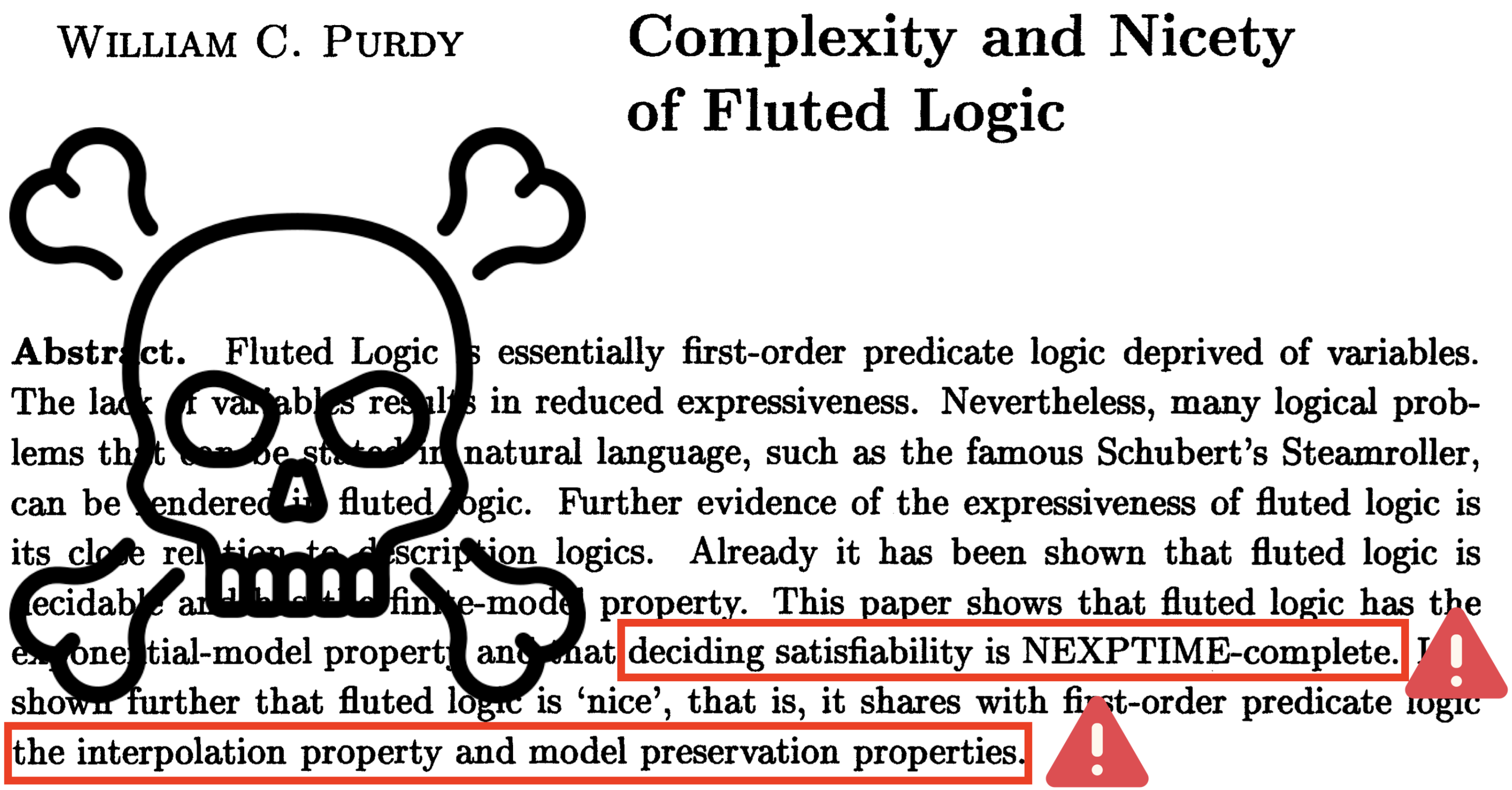
Error 2. L_{suf} does not enjoy CIP (this work!)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Fact: ŁPTP proof of Purdy lack of mathematical arguments.

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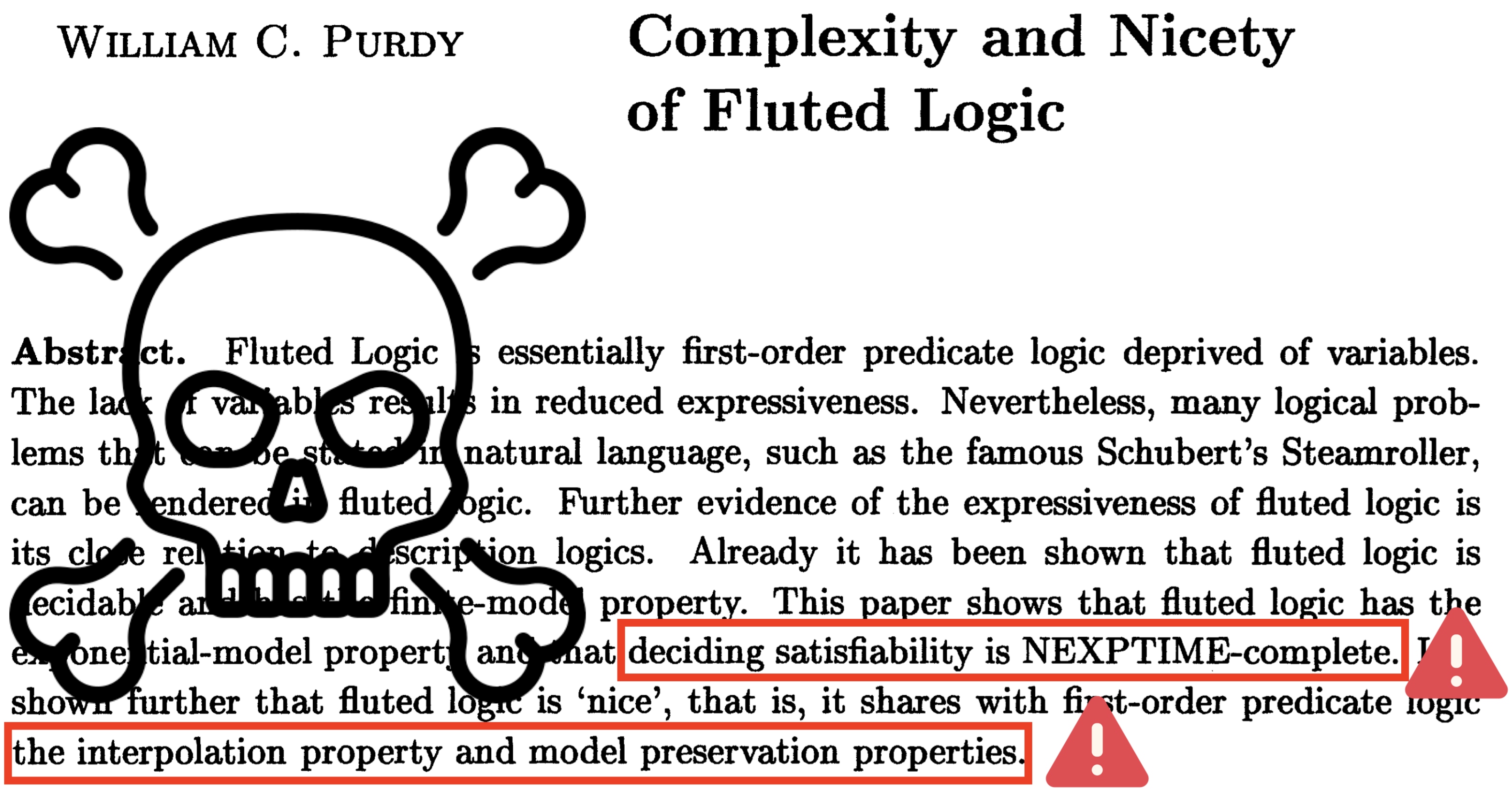

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Conclusion:

On the infamous work of Purdy

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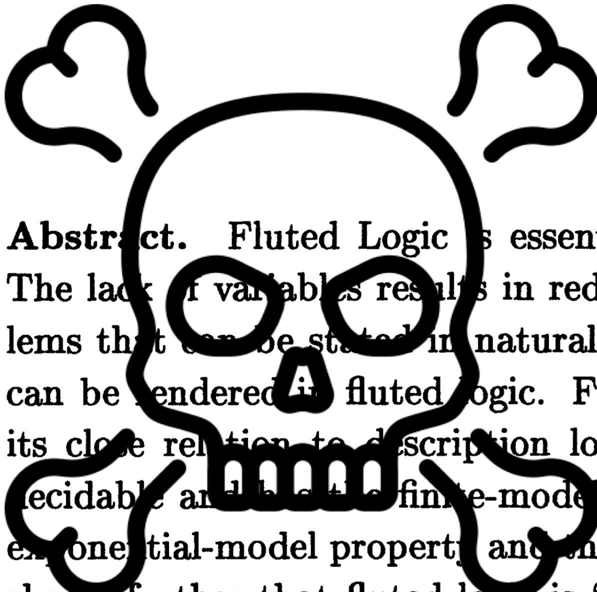
Complexity and Nicety of Fluted Logic



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Conclusion:

We need to study ordered logics more!

Our contribution (Part I)

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We study $\mathsf{L}_{\mathsf{pre}}, \mathsf{L}_{\mathsf{suf}}, \mathsf{L}_{\mathsf{inf}}$ and their guarded subfragments $\mathsf{G}_{\mathsf{pre}}, \mathsf{G}_{\mathsf{suf}}, \mathsf{G}_{\mathsf{inf}}$.

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We study $\mathsf{L}_{\mathsf{pre}}, \mathsf{L}_{\mathsf{suf}}, \mathsf{L}_{\mathsf{inf}}$ and their guarded subfragments $\mathsf{G}_{\mathsf{pre}}, \mathsf{G}_{\mathsf{suf}}, \mathsf{G}_{\mathsf{inf}}$.

1. We introduced a suitable notion of bisimulation.

Our contribution (Part I)

We study $L_{pre}, L_{suf}, L_{inf}$ and their guarded subfragments $G_{pre}, G_{suf}, G_{inf}$.

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2. Comparison of relative expressive powers

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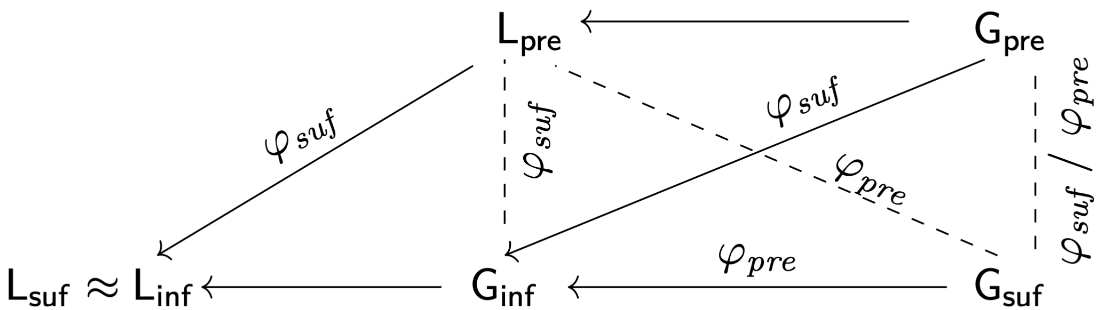

Solid: more expr. Dashed: incomp.

$$\varphi_{pre} := \forall x_1 \forall x_2 \forall x_3\ \mathrm{R}(x_1, x_2, x_3) \rightarrow \mathrm{S}(x_1, x_2)$$

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1. We introduced a suitable notion of bisimulation.
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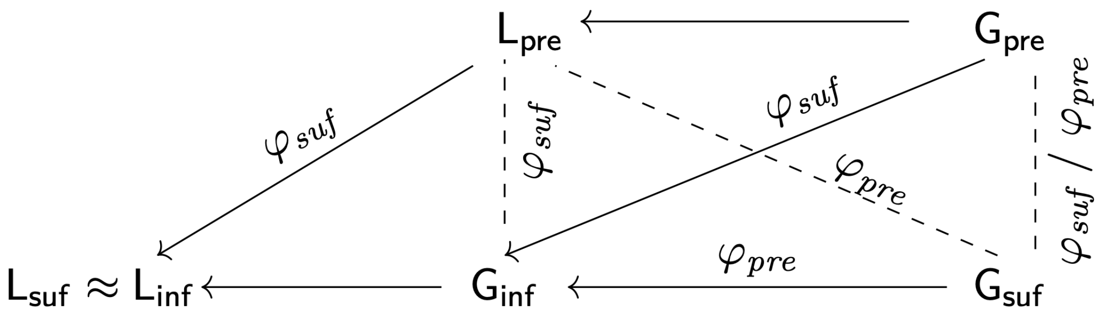

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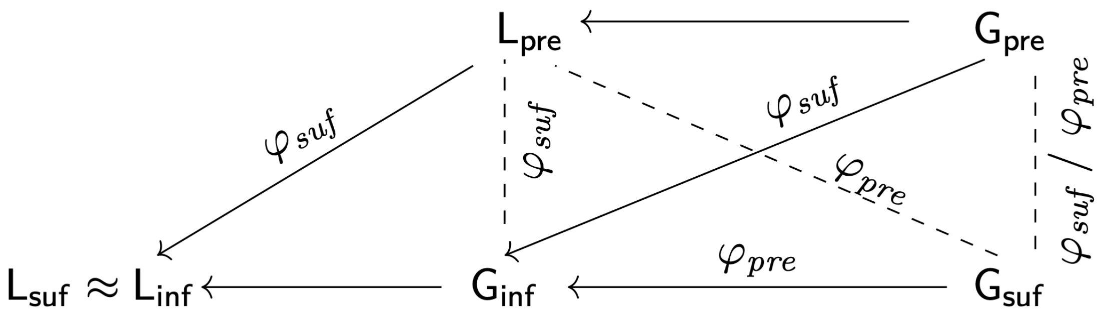

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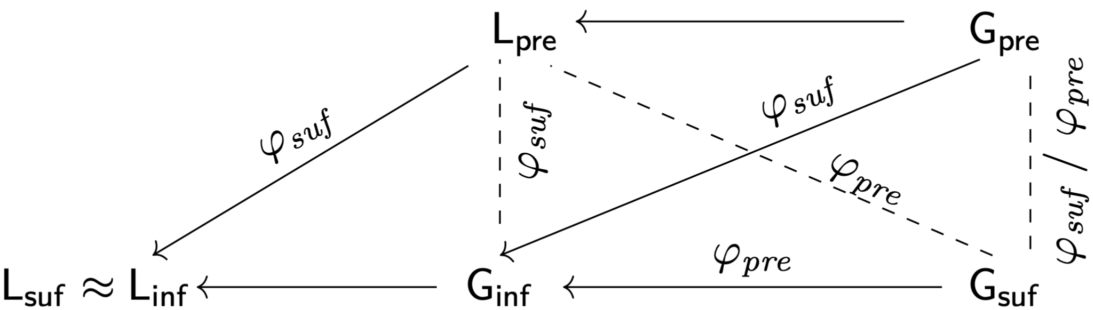

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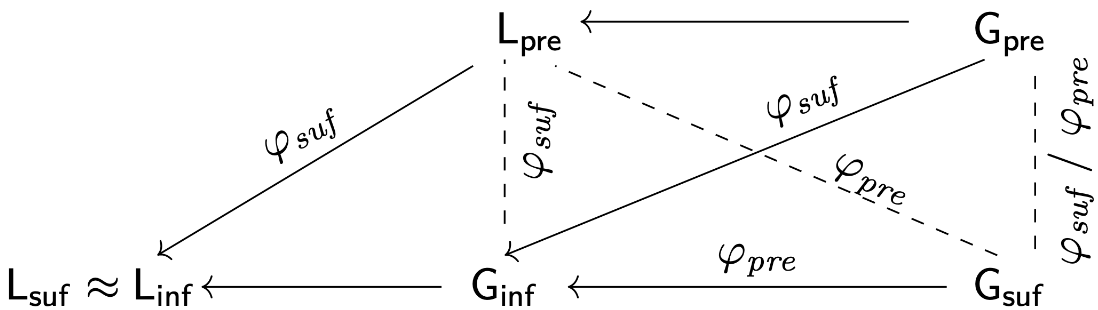

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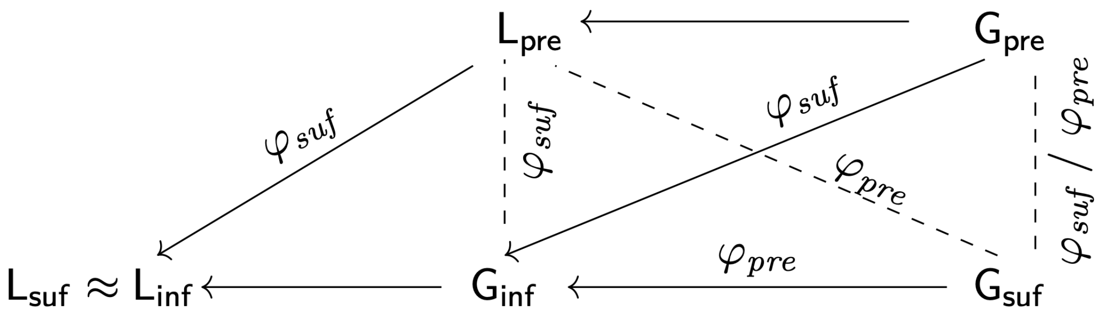

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$$
\begin{tikzcd}
 & \mathsf{L}_{\mathsf{pre}} \arrow[dl, "\varphi_{suf}"'] \arrow[d, dash, dashed, "\varphi_{suf}"] \arrow[dr, dash, dashed, "\varphi_{pre}"] & \mathsf{G}_{\mathsf{pre}} \arrow[l] \arrow[dl, "\varphi_{suf}"'] \arrow[d, dash, dashed, "\varphi_{suf} / \varphi_{pre}"] \\
\mathsf{L}_{\mathsf{suf}} \approx \mathsf{L}_{\mathsf{inf}} & \mathsf{G}_{\mathsf{inf}} \arrow[l] & \mathsf{G}_{\mathsf{suf}} \arrow[l, "\varphi_{pre}"']
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$$\psi := \exists x_1 \exists x_2 \exists x_3 [\mathrm{R}(x_1, x_2) \wedge \mathrm{R}(x_2, x_3) \wedge \mathrm{Q}_1(x_1) \wedge \mathrm{Q}_2(x_2)]$$

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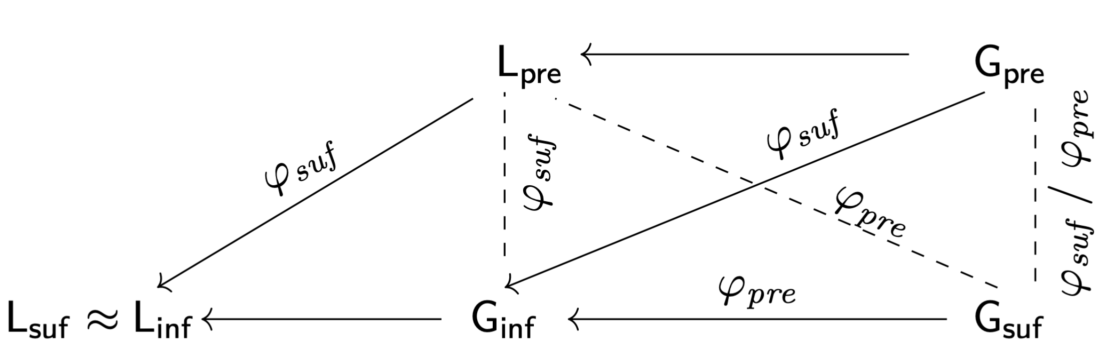

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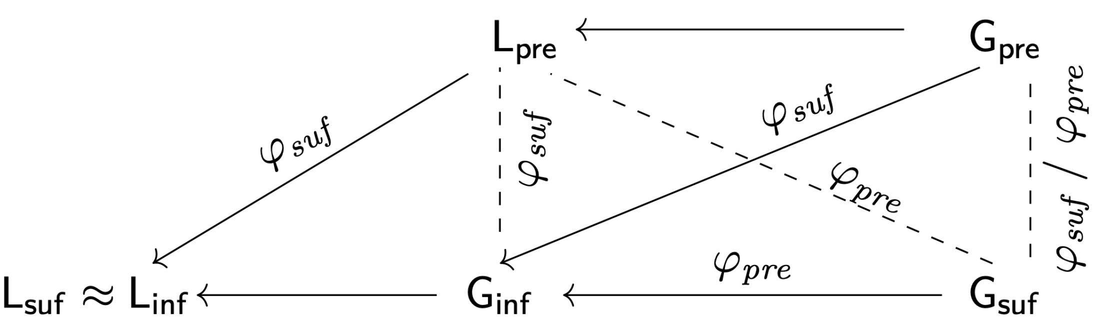

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$\varphi \models \neg\psi$ (why?)

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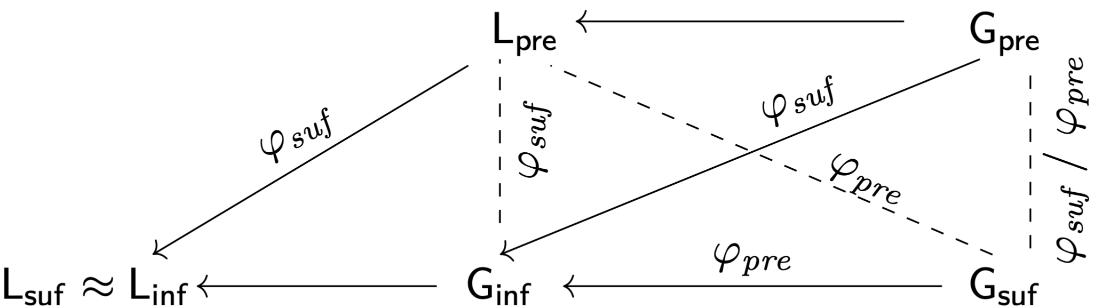

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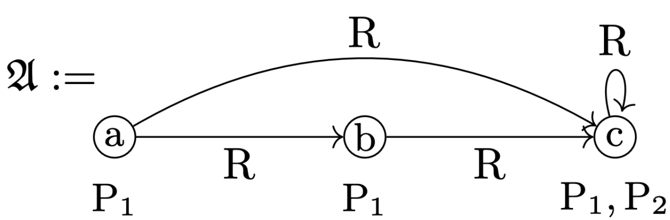
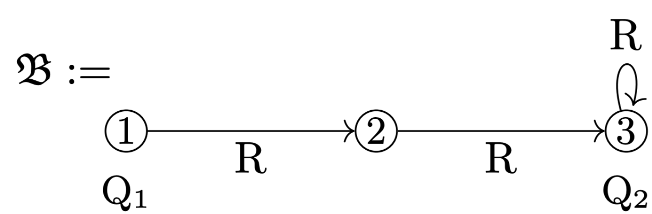
$$\varphi := \forall x_1 \forall x_2 \forall x_3\ [\mathrm{R}(x_1, x_2) \land \mathrm{R}(x_2, x_3) \to (\mathrm{P}_1(x_1) \land \mathrm{P}_2(x_3))] \land \forall x_1 \forall x_2\ [(\mathrm{P}_1(x_1) \land \mathrm{P}_2(x_3)) \to \mathrm{R}(x_1, x_2)]$$

$$\psi := \exists x_1 \exists x_2 \exists x_3 [\mathrm{R}(x_1, x_2) \land \mathrm{R}(x_2, x_3) \land \mathrm{Q}_1(x_1) \land \mathrm{Q}_2(x_2)] \land \forall x_1 \forall x_2\ [(\mathrm{Q}_1(x_1) \land \mathrm{Q}_2(x_2)) \to \neg \mathrm{R}(x_1, x_2)]$$

$\varphi \models \neg\psi$ (why?)

but $\mathfrak{A} \models \varphi$ and $\mathfrak{B} \models \psi$ are





Our contribution (Part I)

We study $\mathsf{L}_{\mathsf{pre}}, \mathsf{L}_{\mathsf{suf}}, \mathsf{L}_{\mathsf{inf}}$ and their guarded subfragments $\mathsf{G}_{\mathsf{pre}}, \mathsf{G}_{\mathsf{suf}}, \mathsf{G}_{\mathsf{inf}}$.

1. We introduced a suitable notion of bisimulation.

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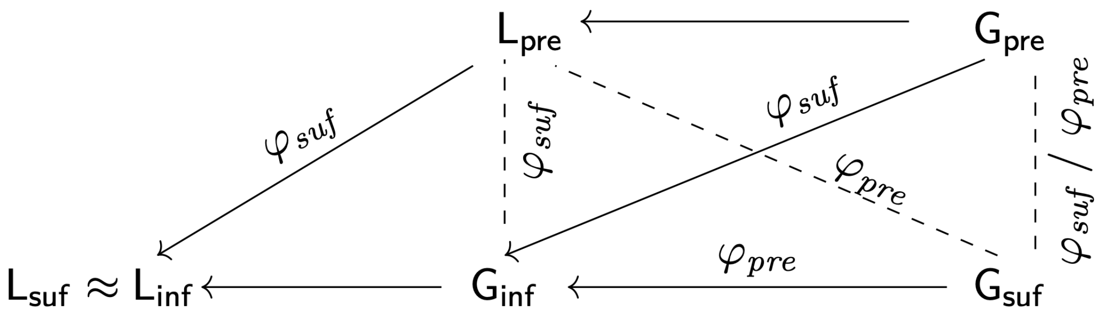

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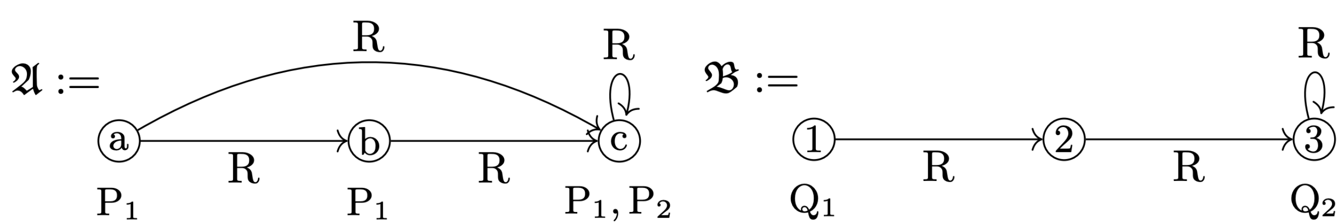

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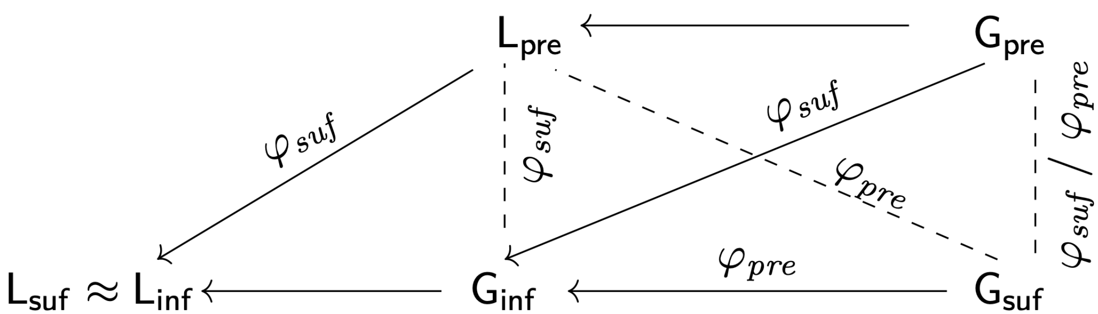

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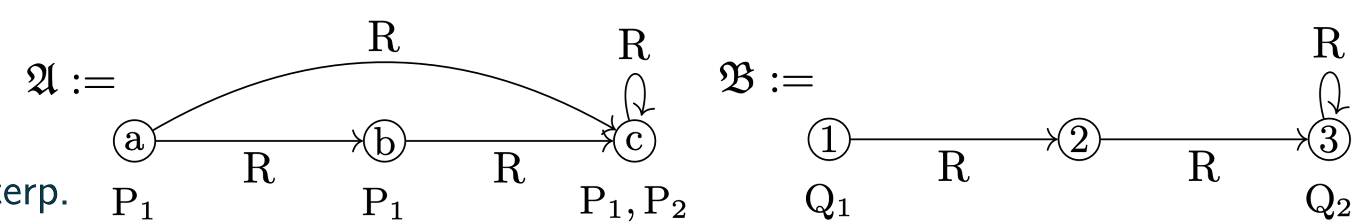

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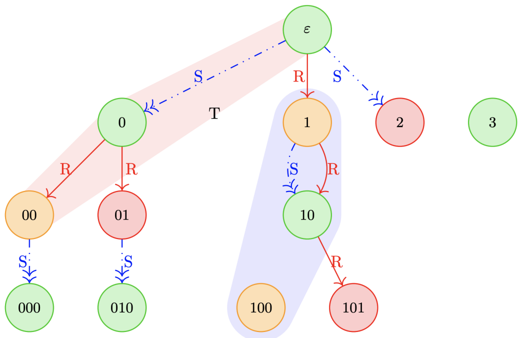

Tree models

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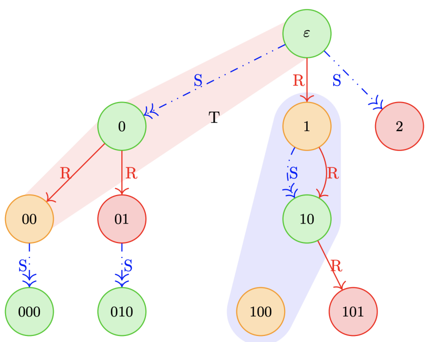

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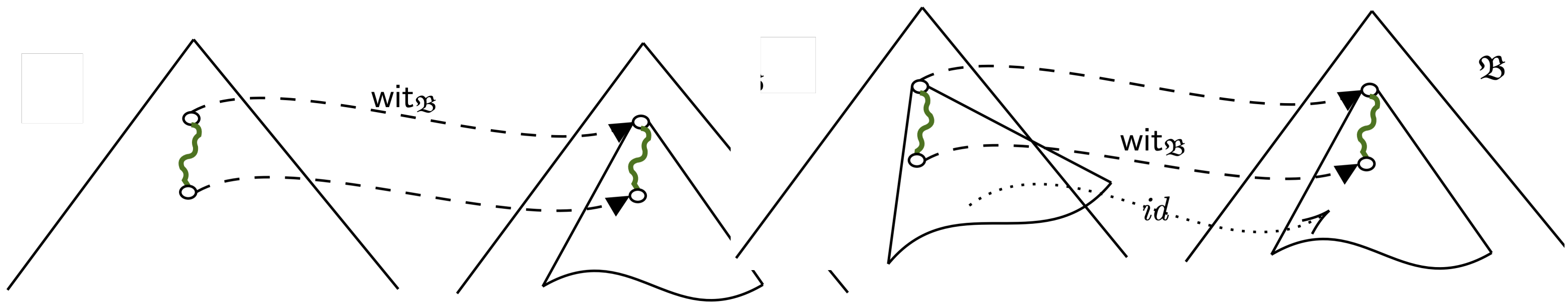

A novel "complete and repair" model construction method

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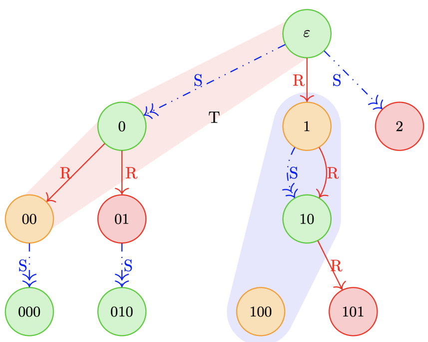

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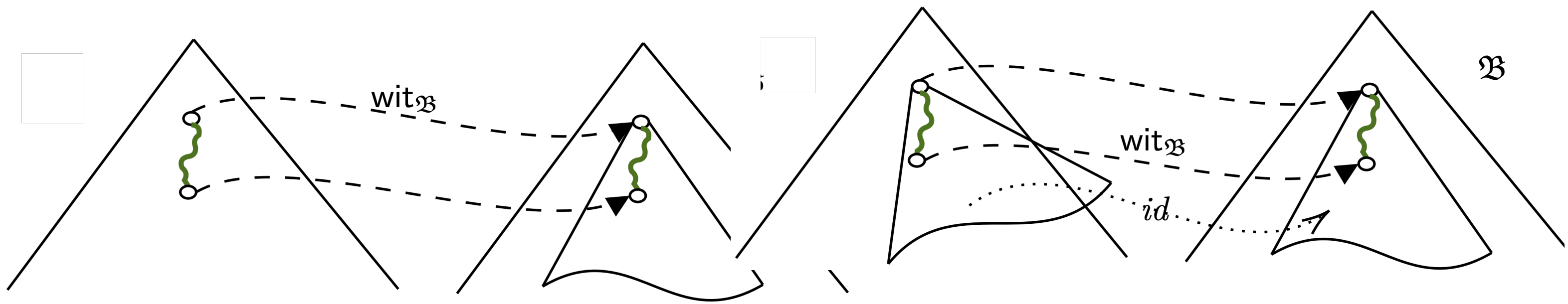

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Works beyond forward $\mathcal{GF}$. A good meta-heuristic.

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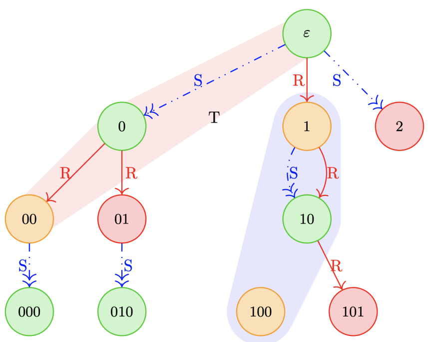

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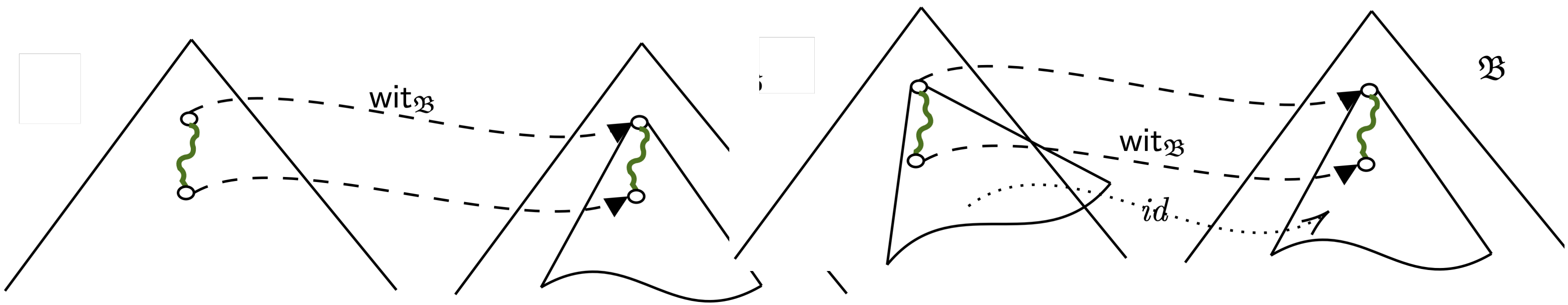

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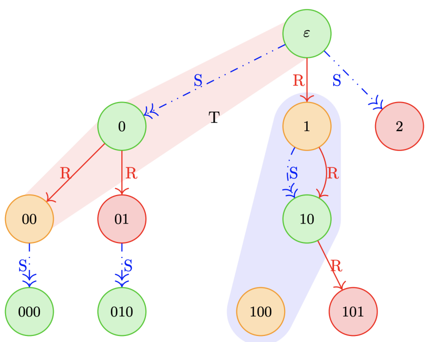

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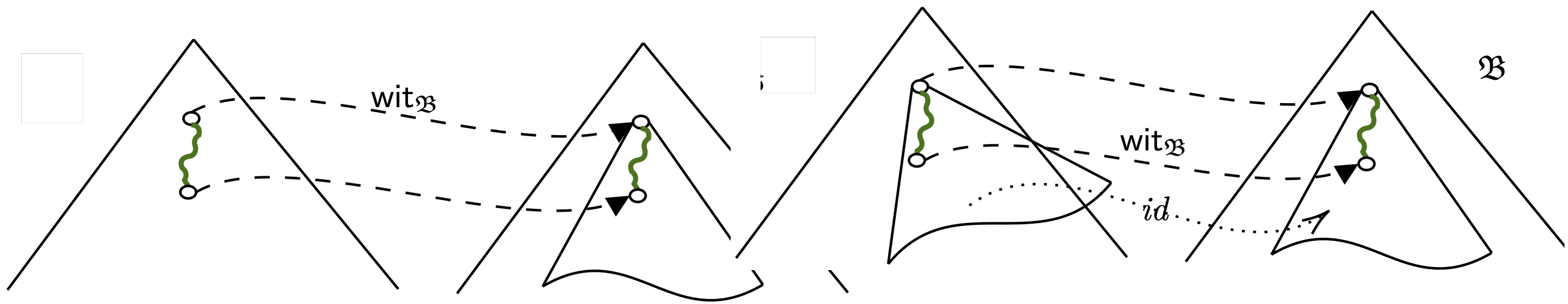

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Conclusions

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1. Status of L_{suf} and L_{inf}

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SAT

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SAT **FMP**

Conclusions

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SAT	FMP	CIP
✓	✓	✗

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SAT
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Open problems: interpolant existence problem, Łoś-Tarski for any equality-free fragment

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Icons that appear in the paper were downloaded from flaticon.com. No changes have been made.