Beyond $\mathcal{ALC}_{\mathsf{reg}}$: Exploring Non-Regular Extensions of PDL with DL Features

4th of September, DL Workshop 2023 & 22nd of September, JELIA 2023

Bartosz “Bart” Bednarczyk

With special thanks to Reijo Jaakkola, Witek Charatonik, and Sebastian Rudolph for all their support.

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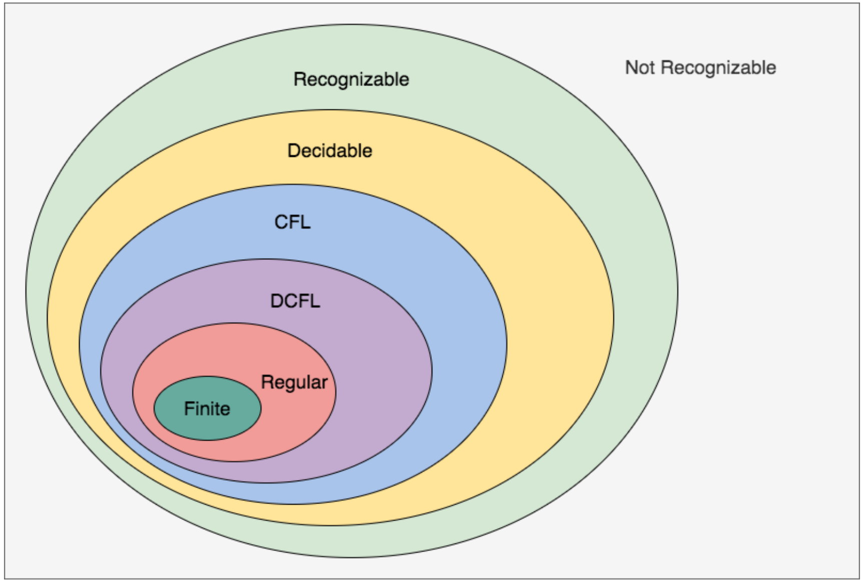

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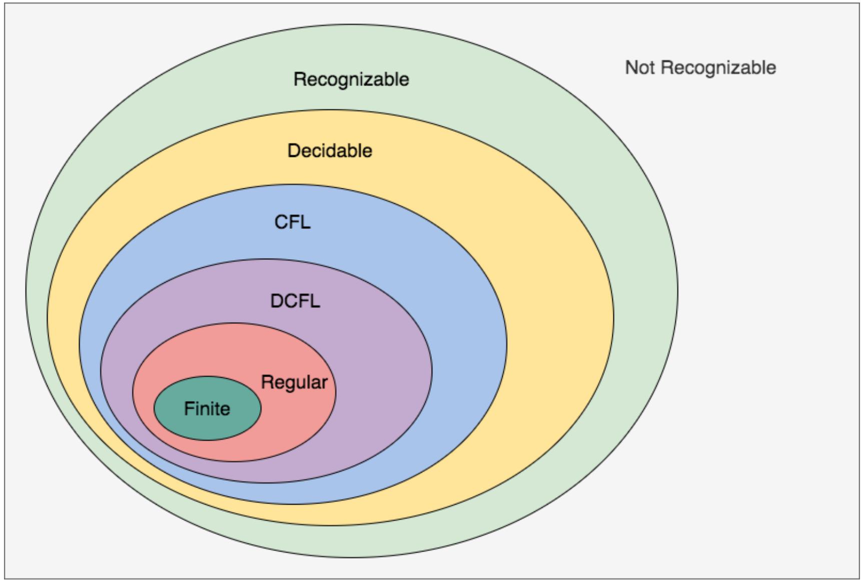

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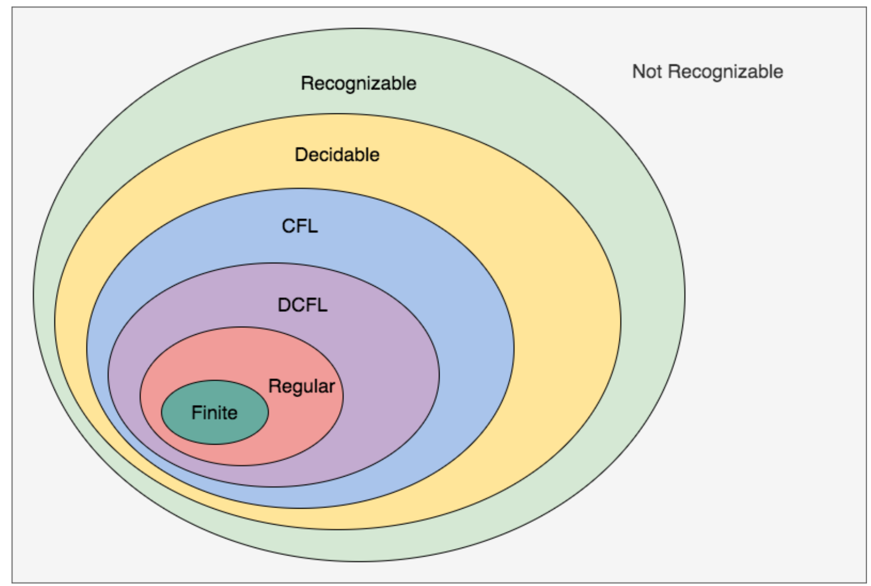

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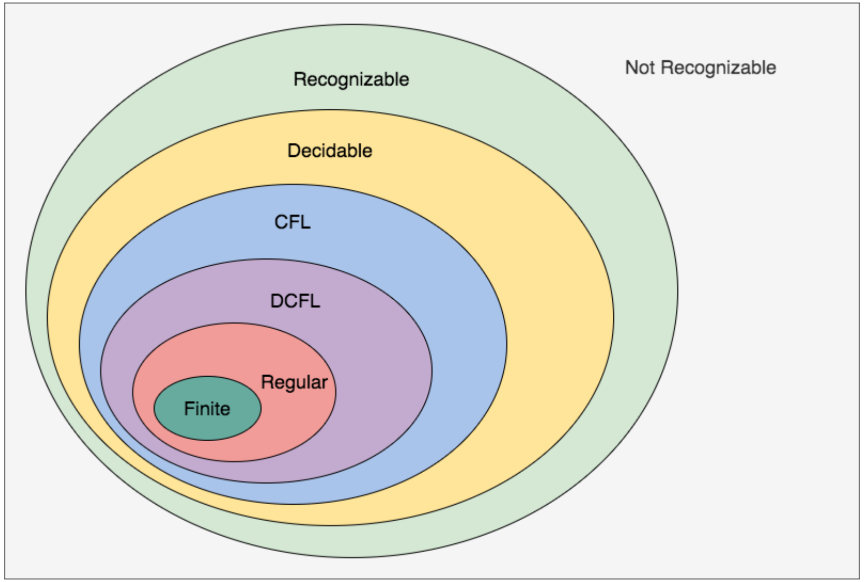

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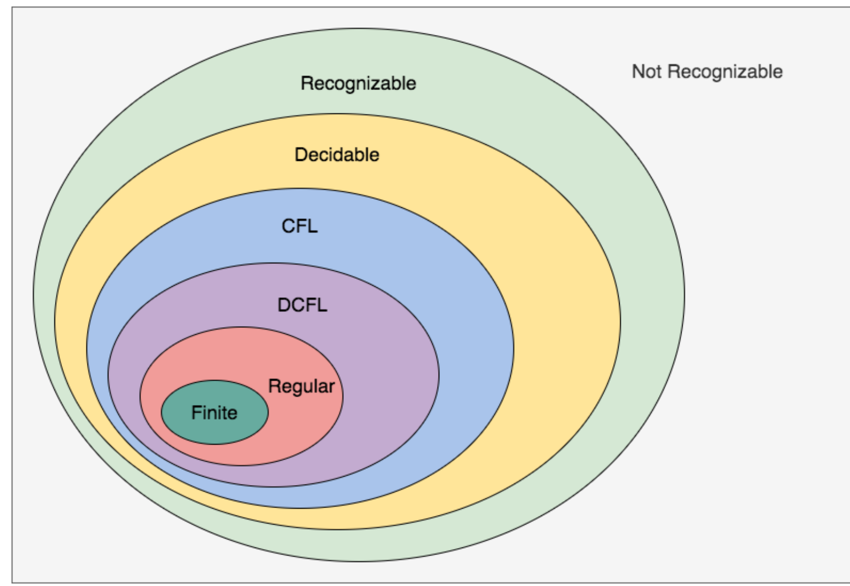

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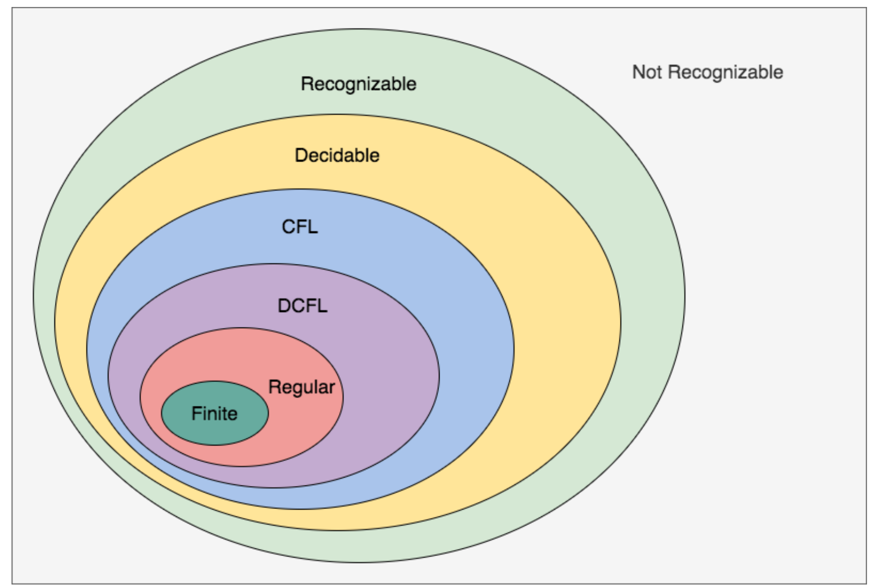

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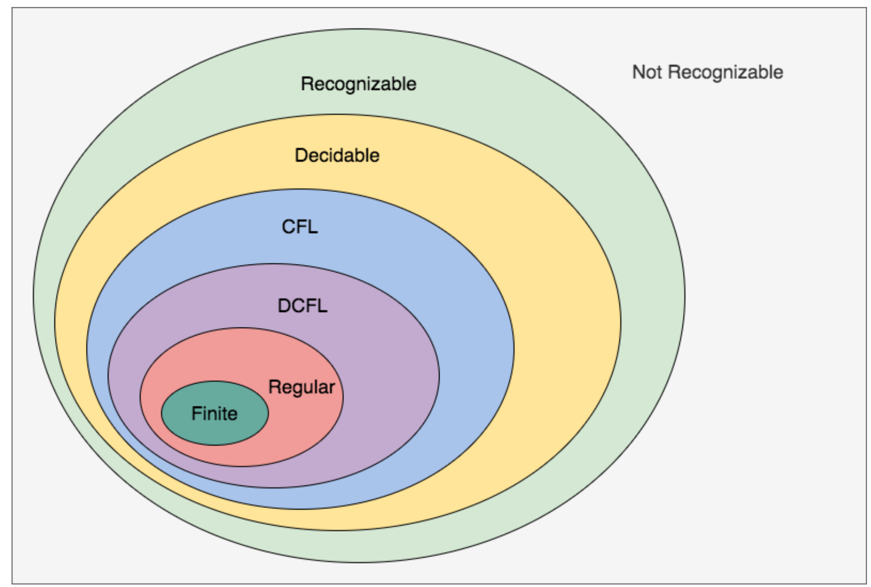

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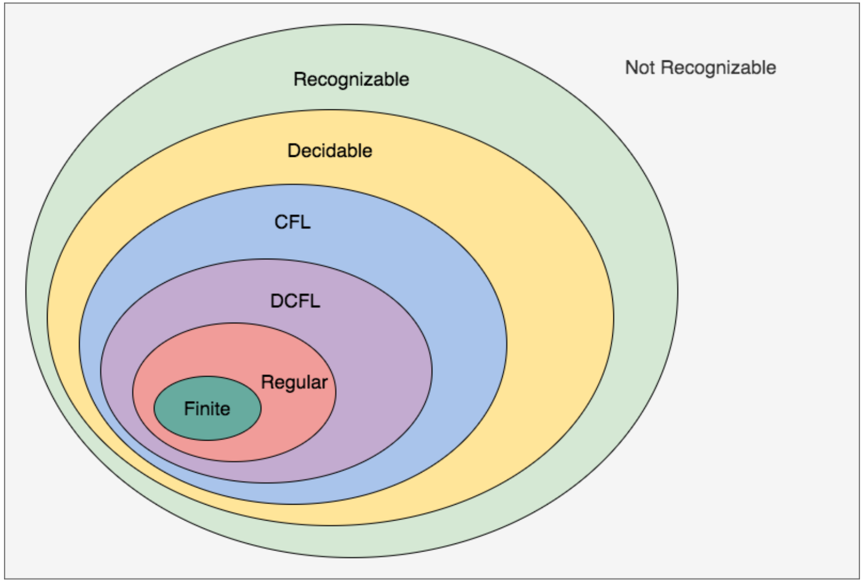

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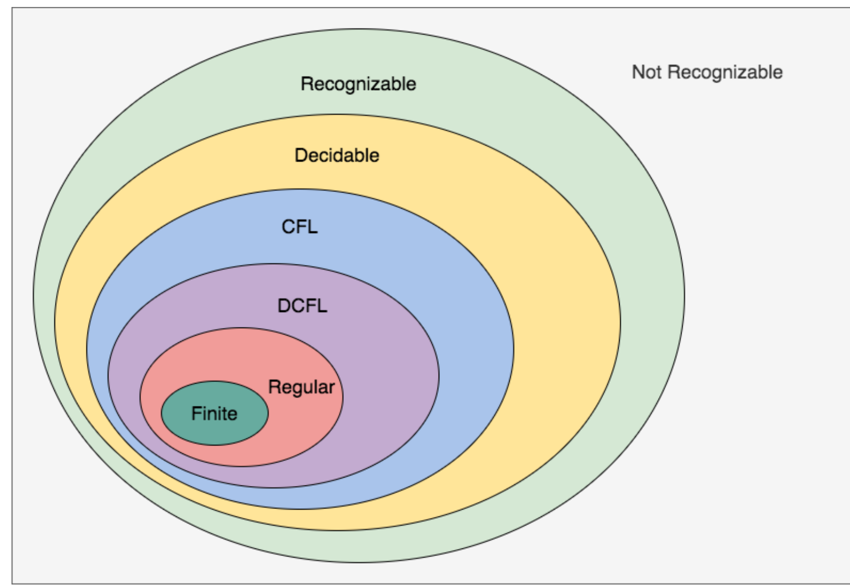
$\mathcal{ALC}$+Visibly Pushdown Languages is decidable.



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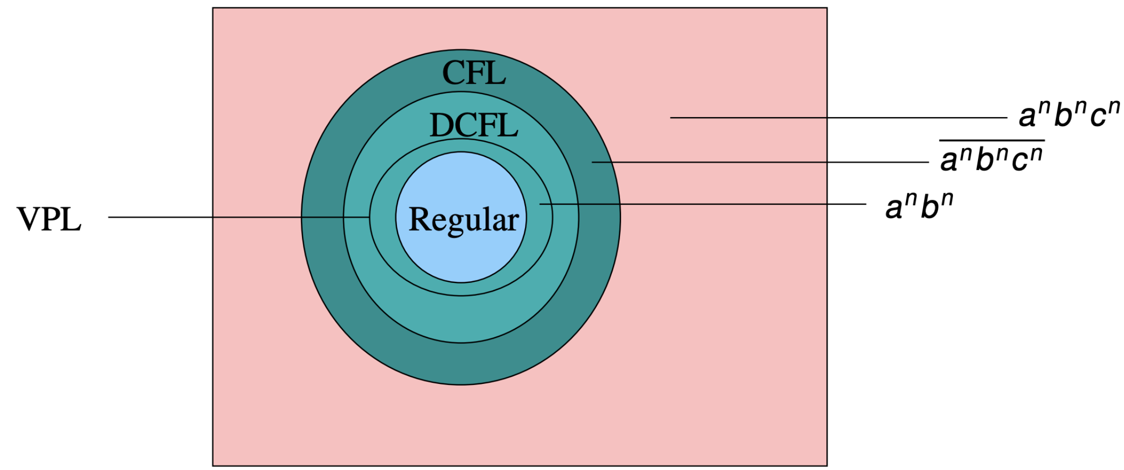

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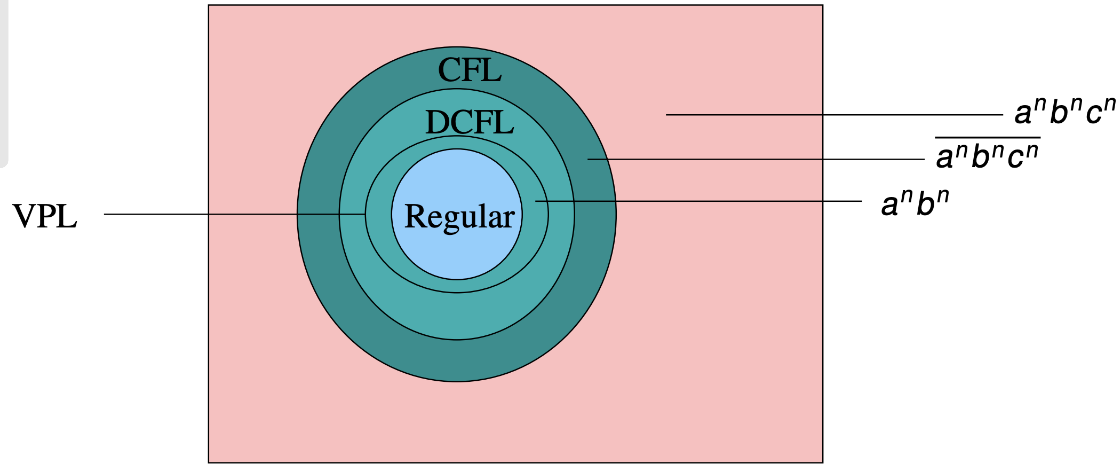

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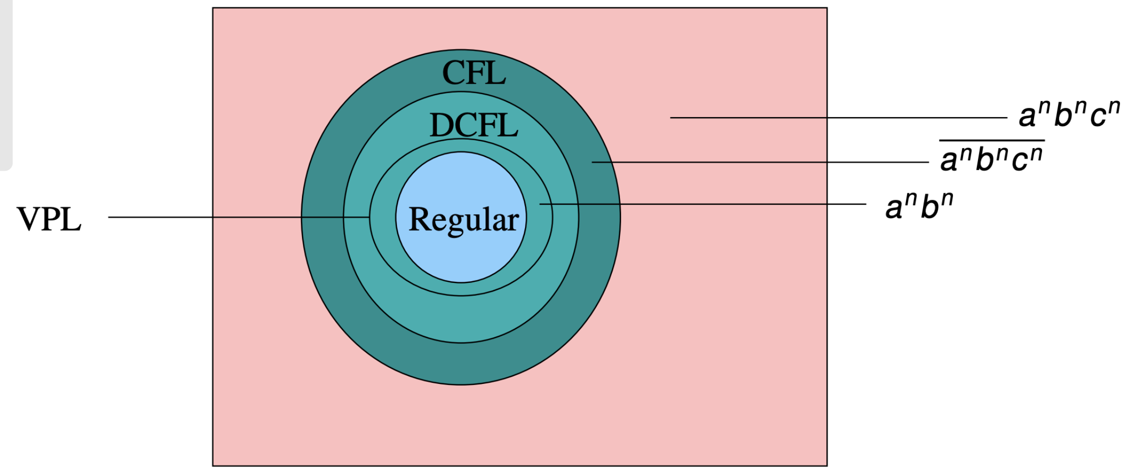

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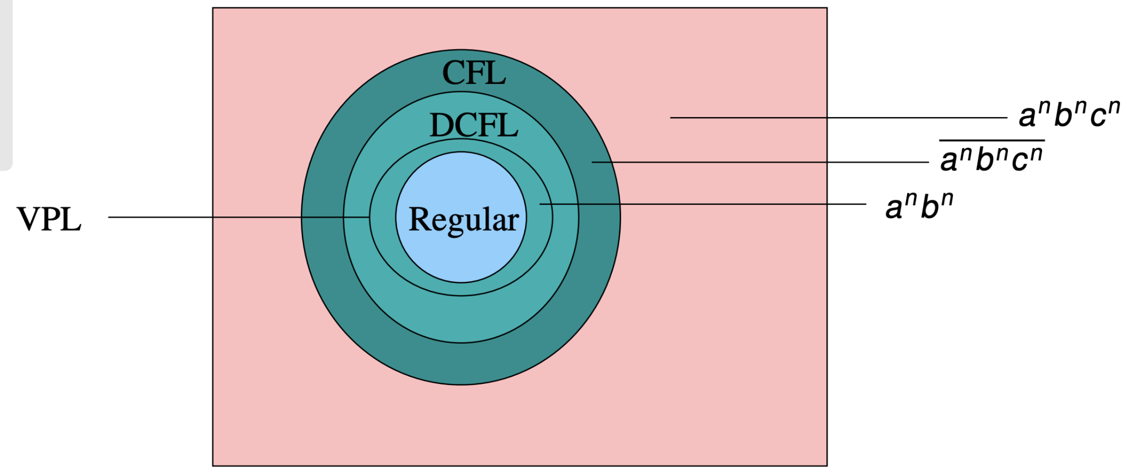

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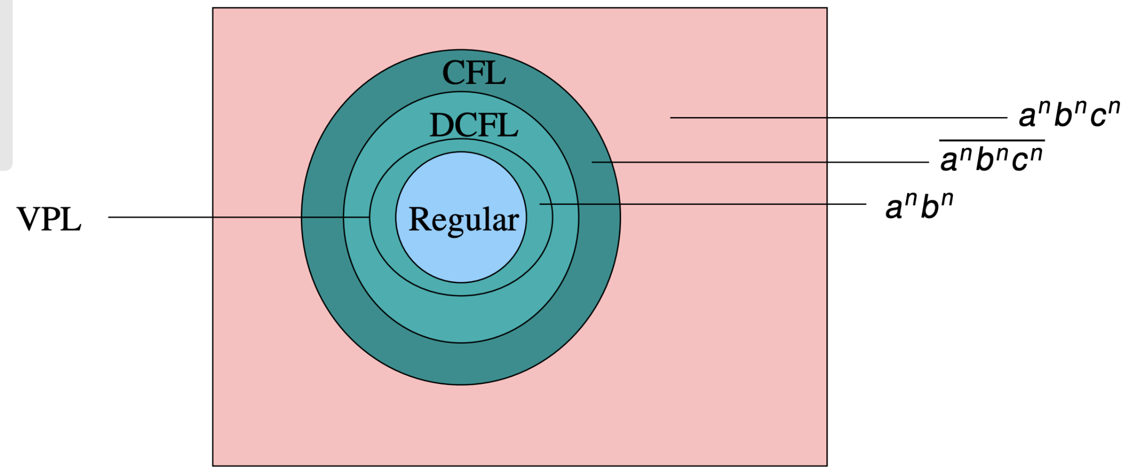

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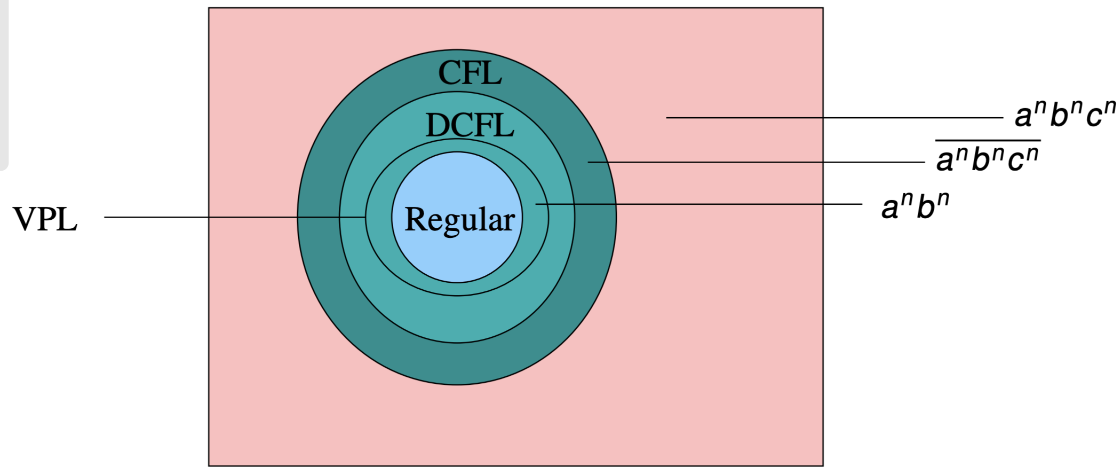

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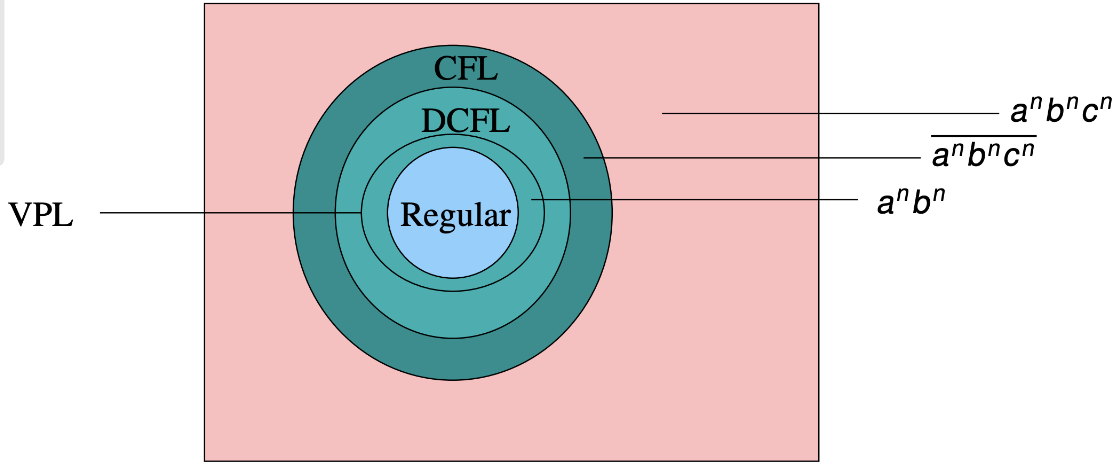

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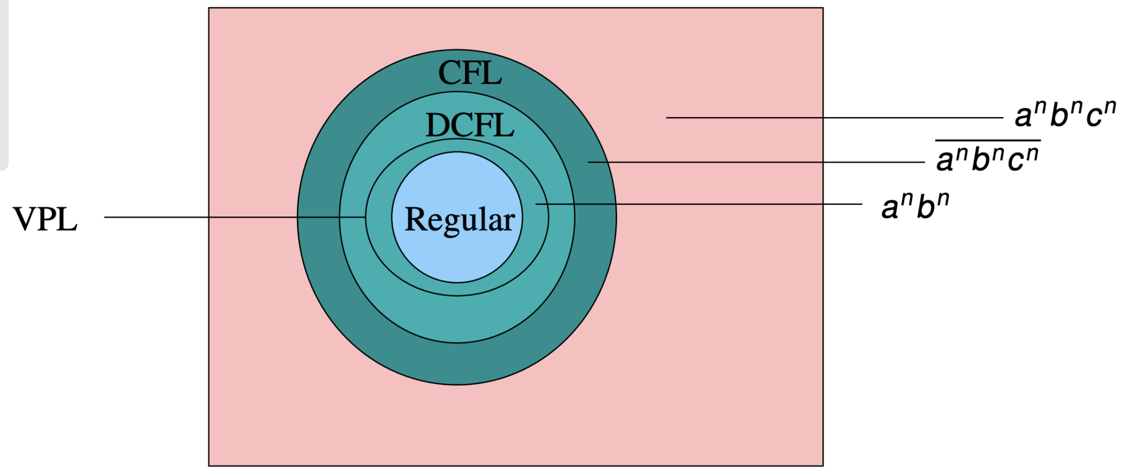

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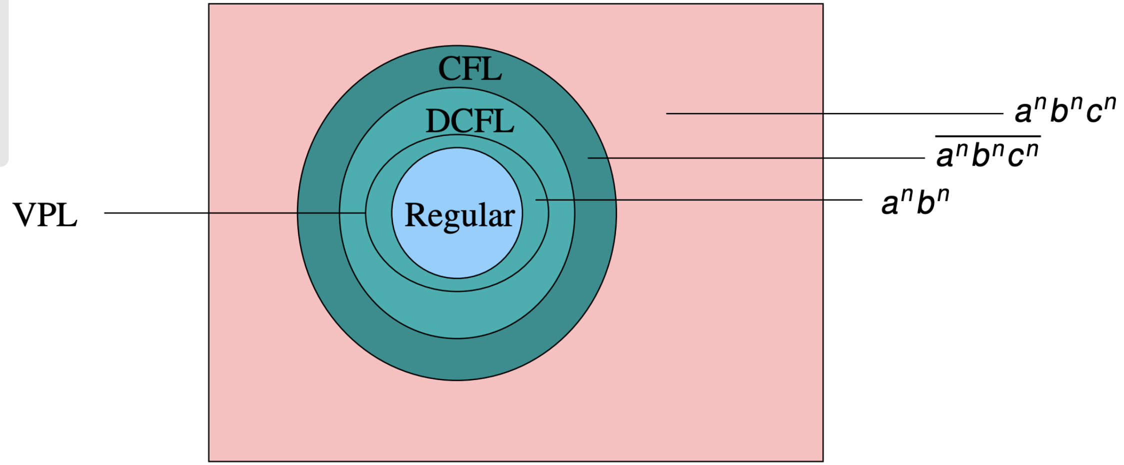

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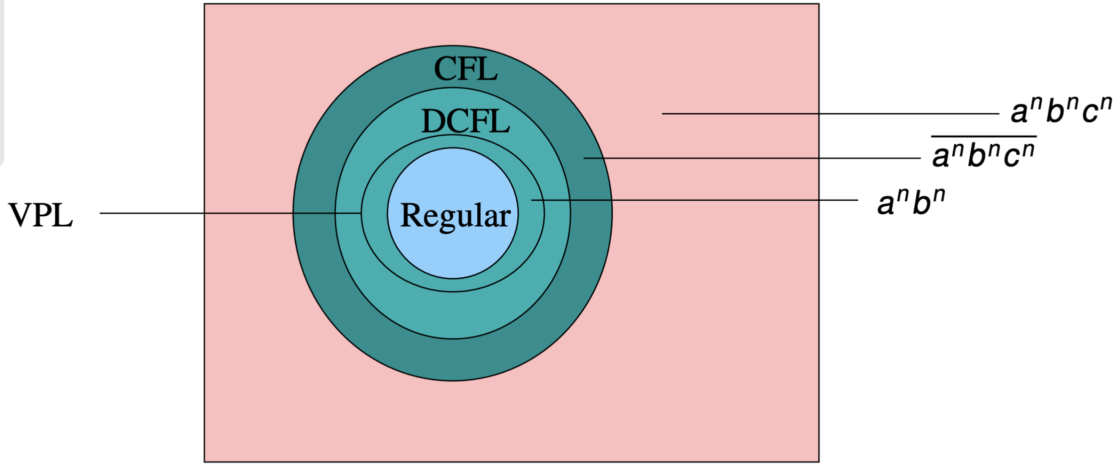

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	Emptiness	Universality/Equivalence	Inclusion
NFA	NLOGSPACE	PSPACE	PSPACE
PDA	PTIME	Undecidable	Undecidable
DPDA	PTIME	Decidable	Undecidable
VPA	PTIME	EXPTIME	EXPTIME

	Closure under				
	Union	Intersection	Complement	Concat.	Kleene-$*$
Regular	Yes	Yes	Yes	Yes	Yes
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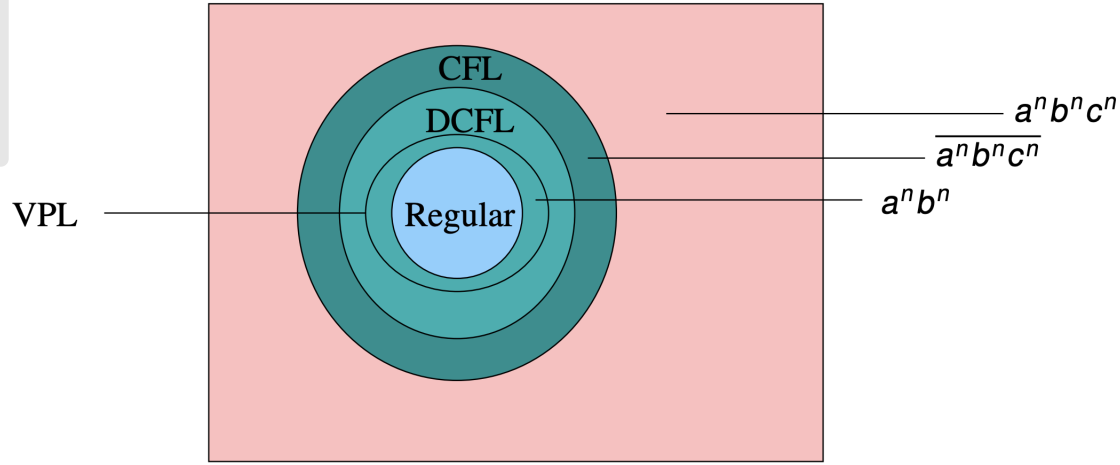

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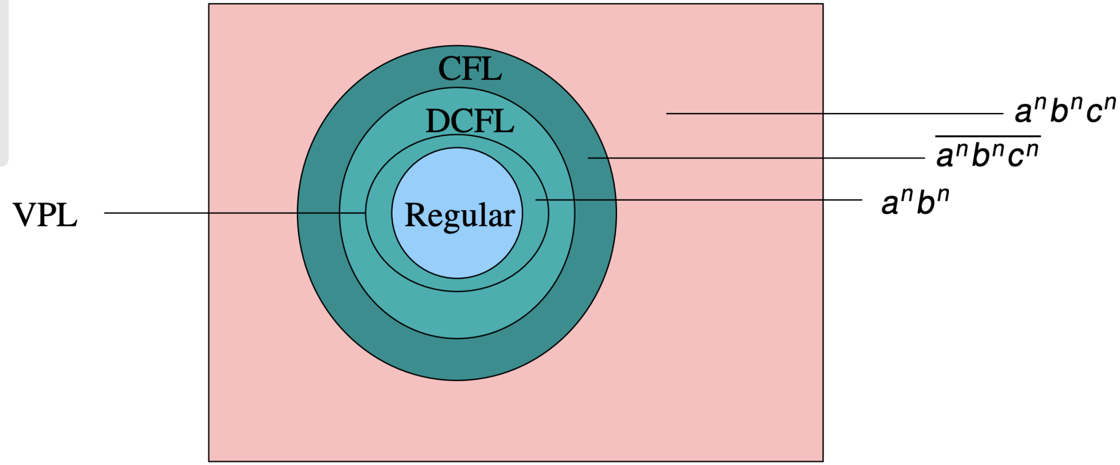

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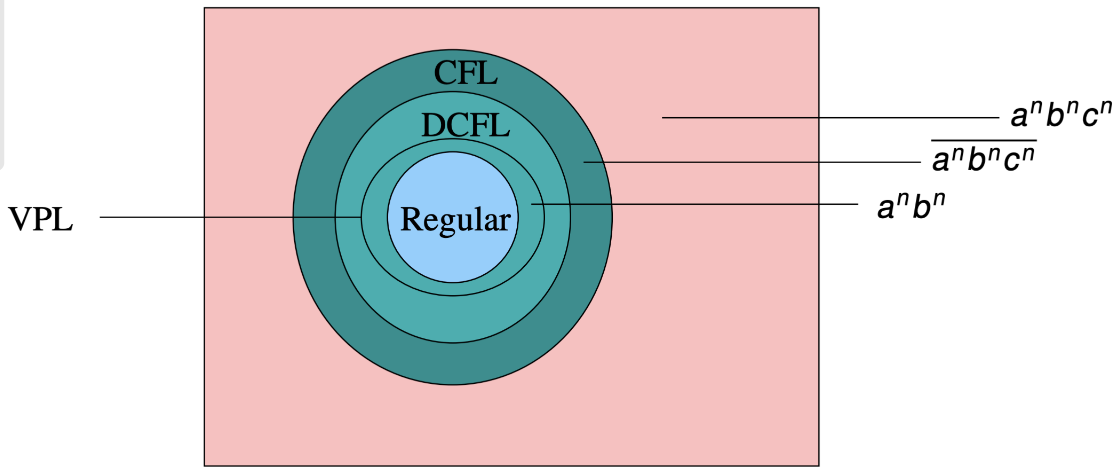

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Why not to employ $\mathbb{VPL}$ in knowledge representation?

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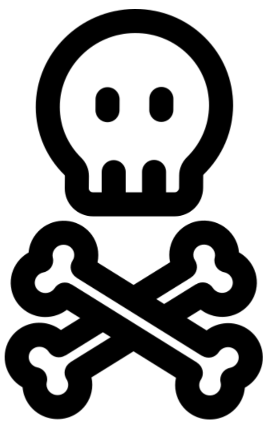

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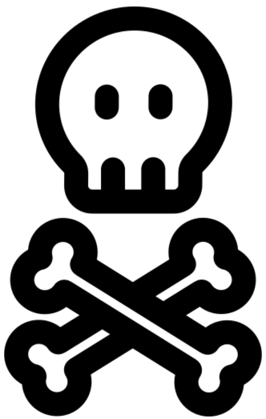

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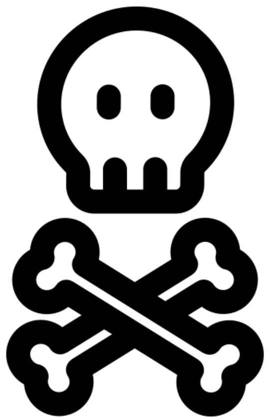

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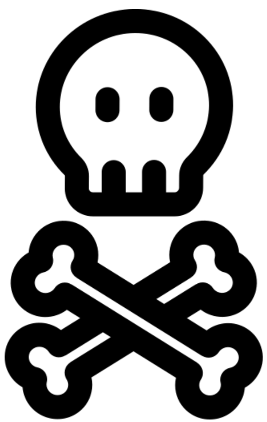
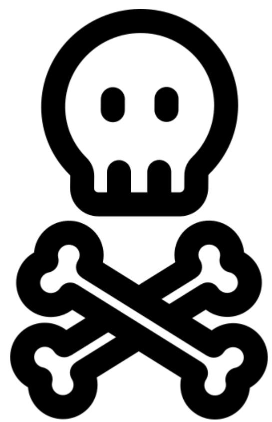
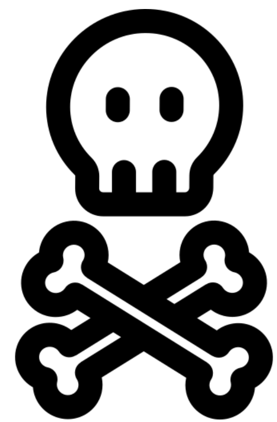
How about other features? How about querying?

Beyond $\mathcal{ALC}_{\mathsf{reg}}$: Exploring Non-Regular Extensions of PDL with Description Logics Features

Bartosz Bednarczyk[1,2]

Loops	Nominals	Queries
		

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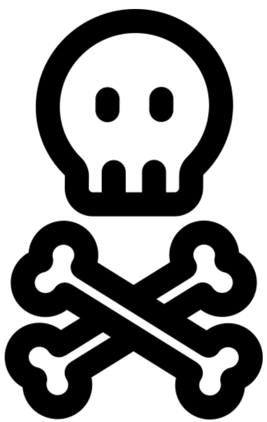
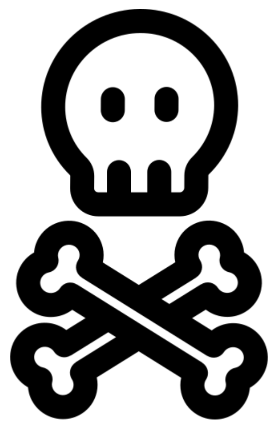


Visibly one counter

Nominals



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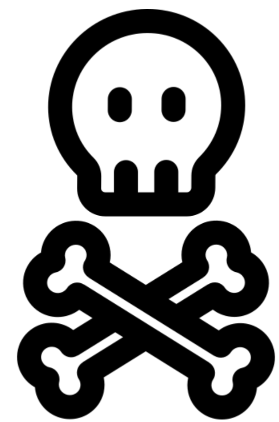

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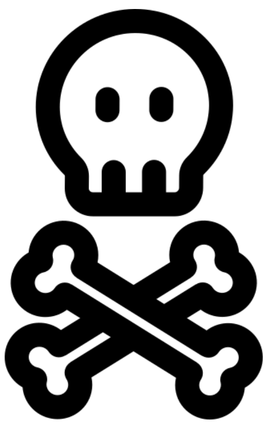

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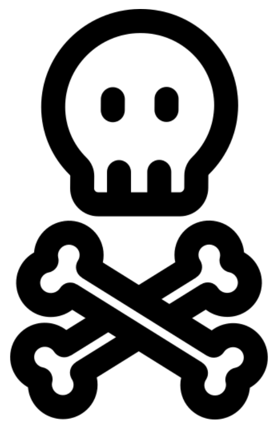

$\mathcal{ALC}^{r\#s\#}_{\mathsf{reg}}$

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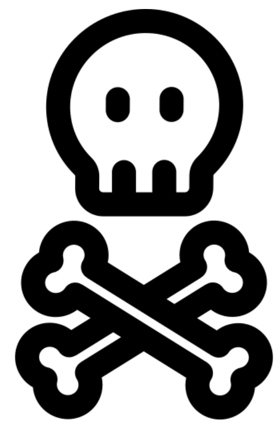

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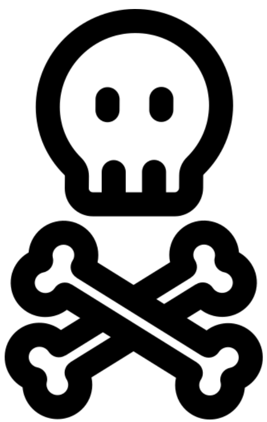

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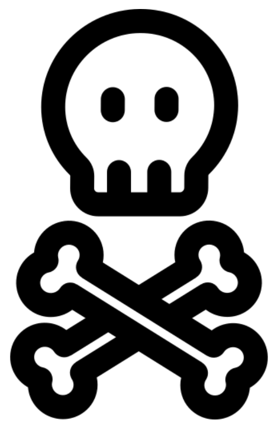

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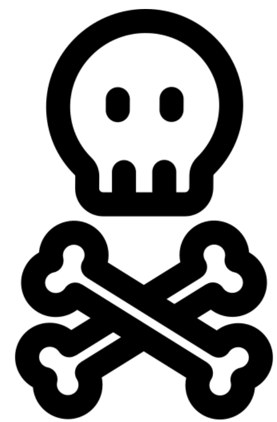

$\mathcal{ALC}$-TBoxes + CRPQs with $r^{\#}s^{\#}$

Proof sketch: Undecidability of $\mathcal{ALC}_{\mathsf{vpl}}$ + Self

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Input: Deterministic one counter automata $\mathcal{A}_1$, $\mathcal{A}_2$.

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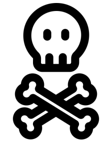

Valiant 1973

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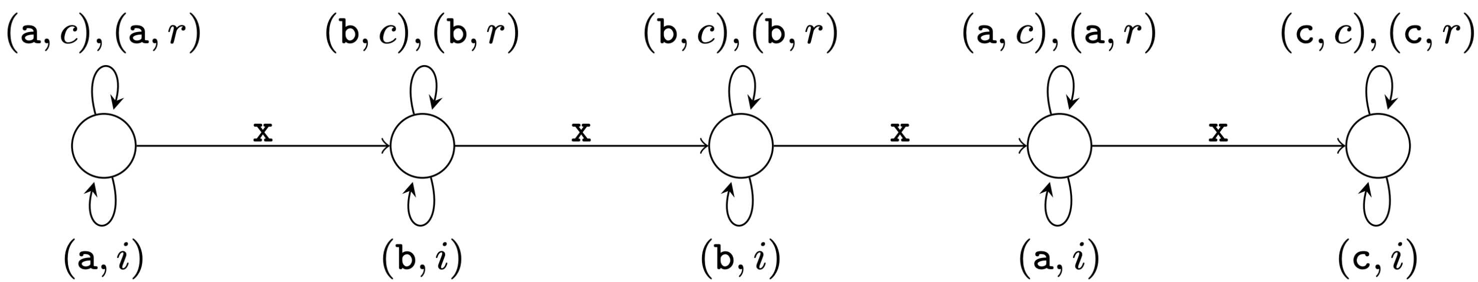

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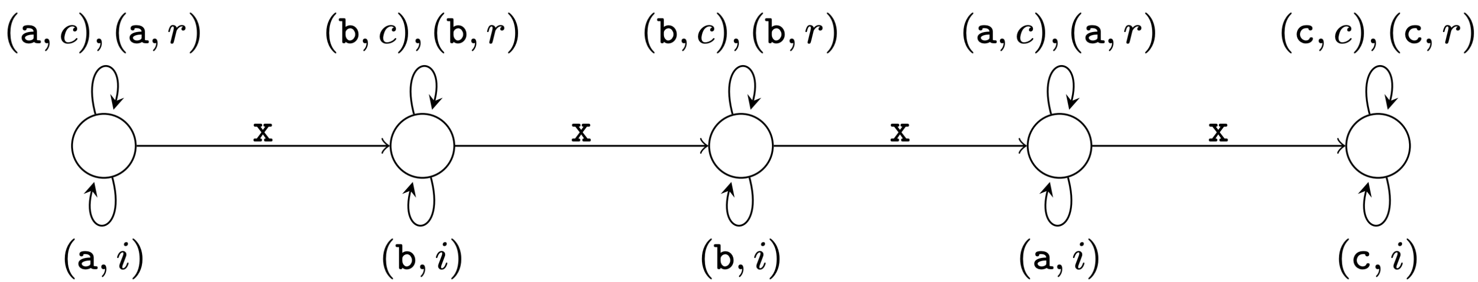

Trick 2: Employ concepts $\forall\hat{\mathcal{A}}_1.\mathrm{OK}_1 \sqcap \forall\hat{\mathcal{C}}_1.\neg\mathrm{OK}_1$ to decorate interpretations with "acceptance" of $\mathcal{A}_1$.

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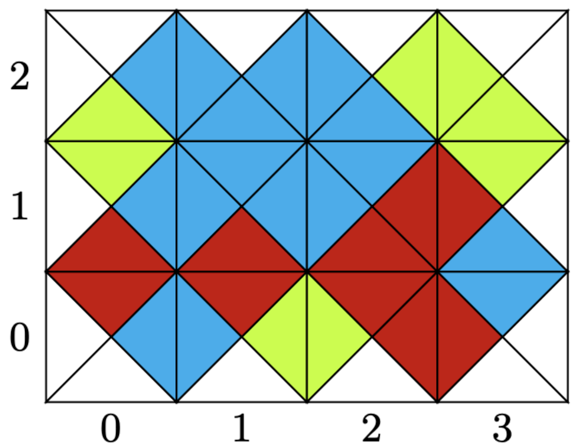
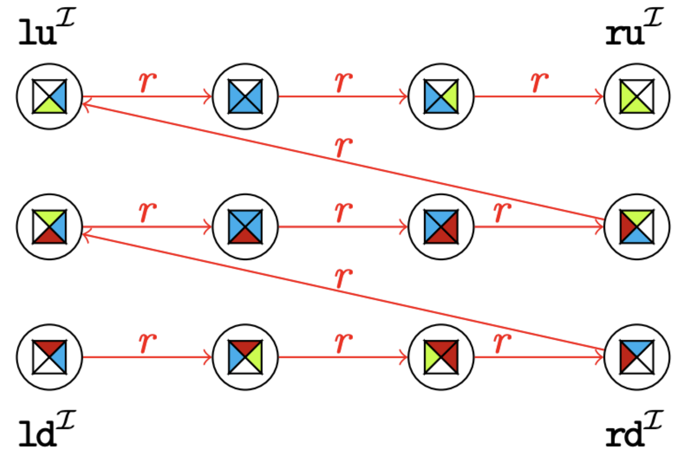


(a) Visualization of ξ.



(b) The encoding of ξ as a $\mathcal{D}$-snake $\mathcal{I}$.

Fig. 1: If $\mathrm{Col} = \{$, , , $\}$ and $\mathrm{T} = \mathrm{Col}^4$, the map $\xi := \{(0,0) \mapsto$, $(1,0) \mapsto$, $(2,0) \mapsto$, $(3,0) \mapsto$, $(0,1) \mapsto$, $(1,1) \mapsto$, $(2,1) \mapsto$, $(3,1) \mapsto$, $(0,2) \mapsto$, $(1,2) \mapsto$, $(2,2) \mapsto$, $(3,2) \mapsto$ $\}$ covers $\mathbb{Z}_4 \times \mathbb{Z}_3$.

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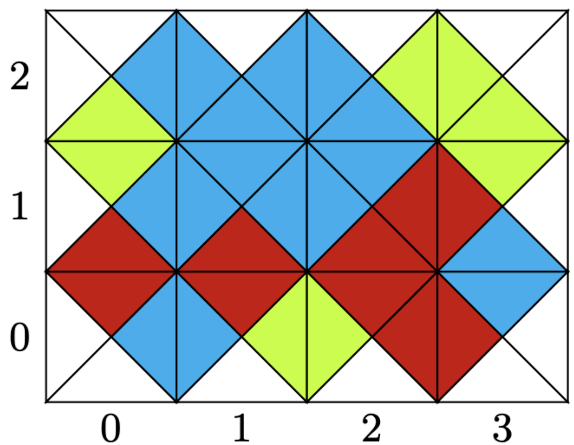

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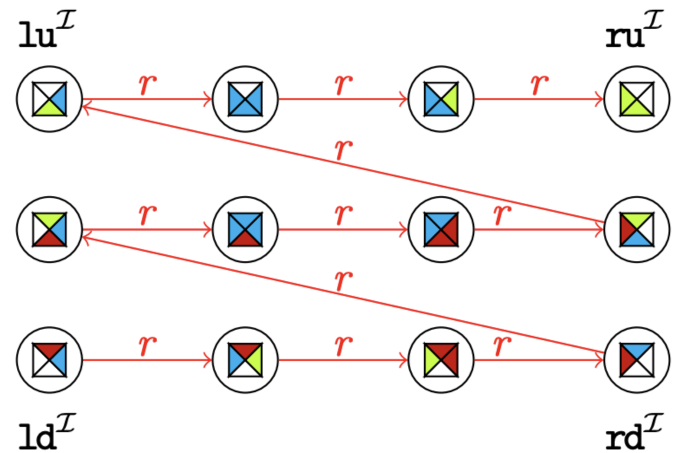

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Problem 1: How to express existence of an N such that every N steps from the start a left-border tile occurs?

Proof sketch: Undecidability of $\mathcal{ALCO} + r^{\#}s^{\#}$ (Introduction)

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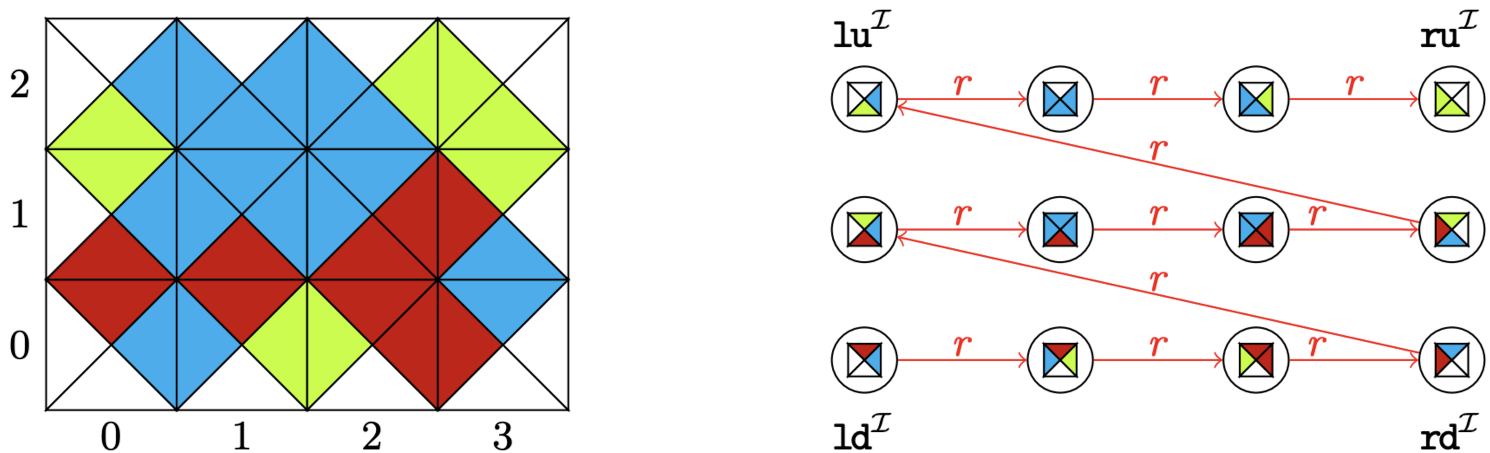

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Problem 1: How to express existence of an N such that every N steps from the start a left-border tile occurs?

Problem 2: How to express that a tile and the tile N steps further have matching sides?

To solve problems from the previous slide, we must teach snakes how to measure. Use yardsticks!

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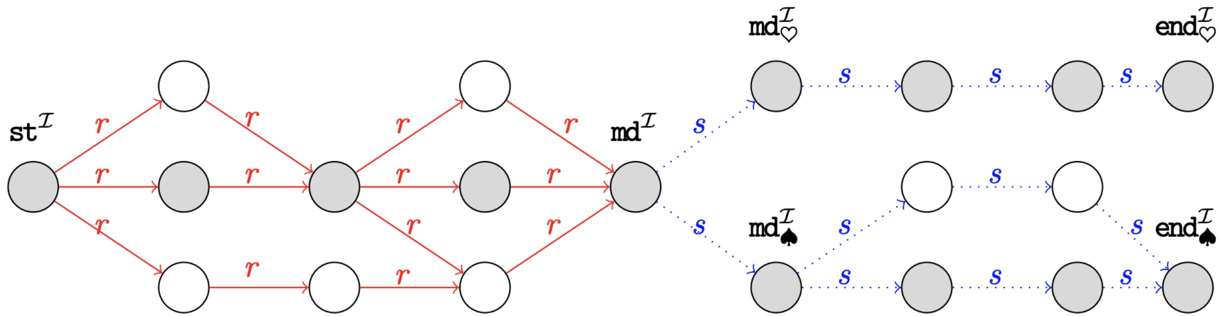

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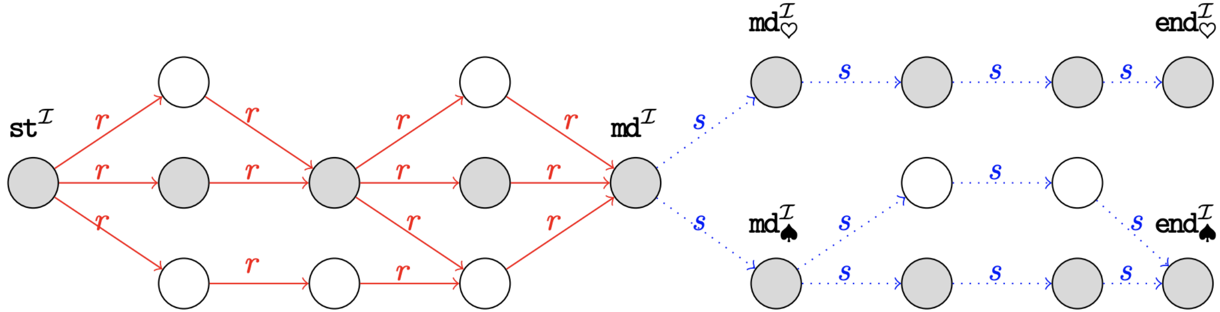

Key property: there is unique N s.t. distances $\mathtt{st} \rightsquigarrow \mathtt{md}$ and $\mathtt{md} \rightsquigarrow \mathtt{end}_{\mathrm{t}}$ are all equal to N.

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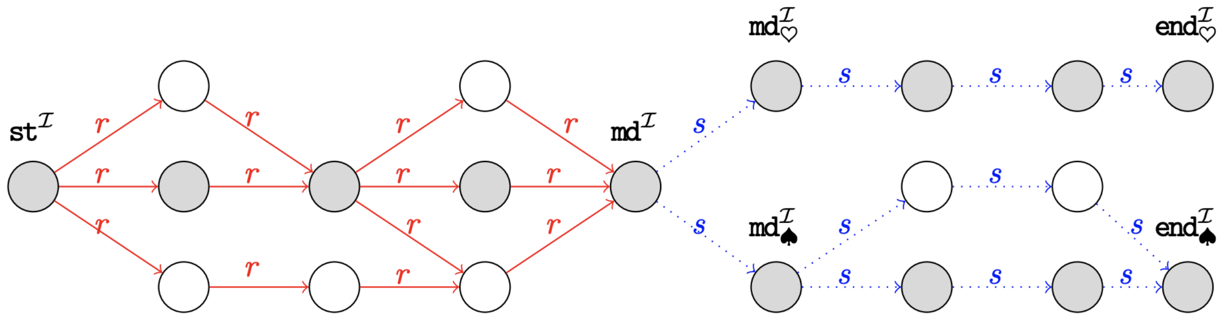

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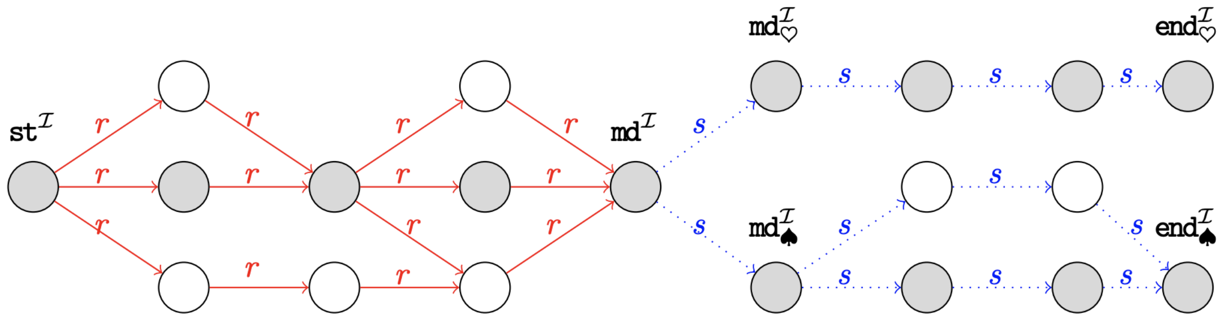

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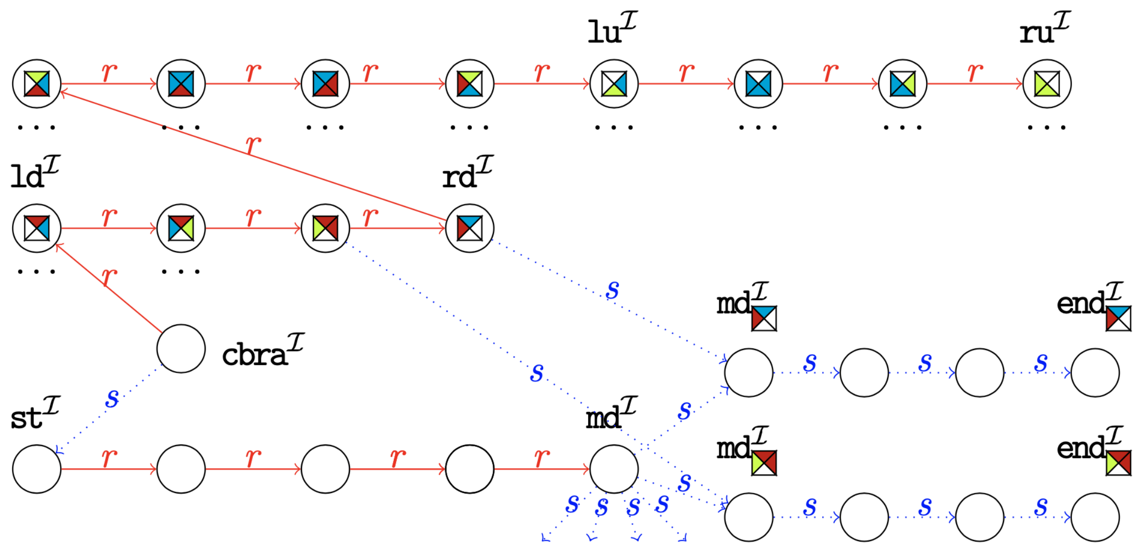

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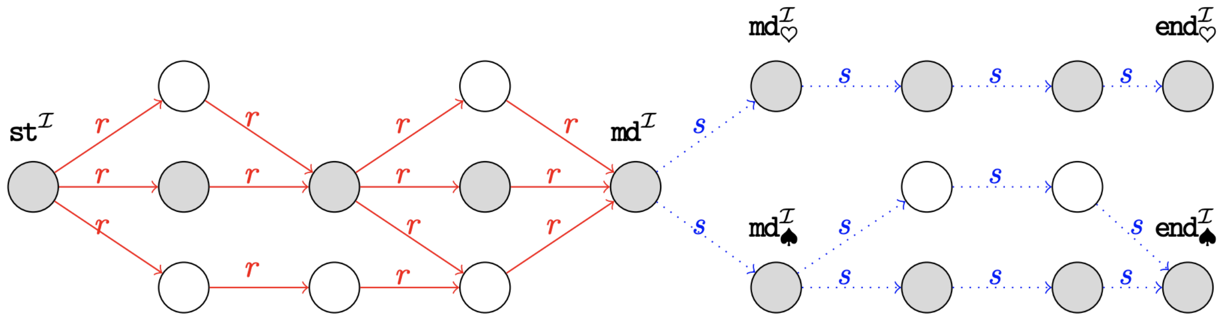

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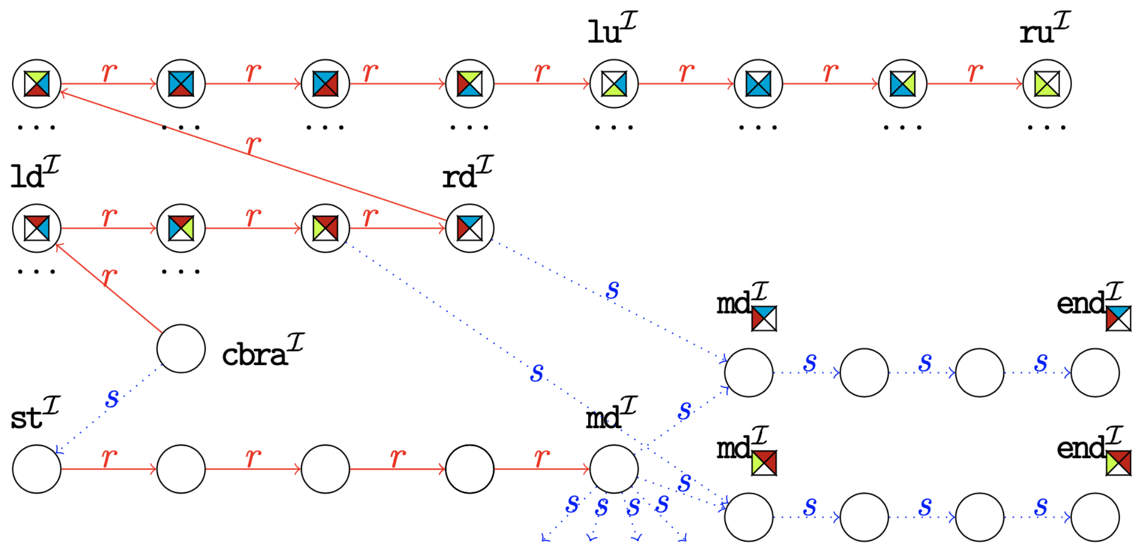
Key property:

An element N steps after d carries a tile t iff d can $r^{\#}s^{\#}$ reach $\mathrm{end}_{\mathrm{t}}$.



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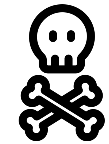
Output: Can we cover an infinite triangle (a.k.a. octant) according to tiling rules?



Proof sketch: Undecidability of querying $\mathcal{ALC}$-TBoxes with non-regular queries

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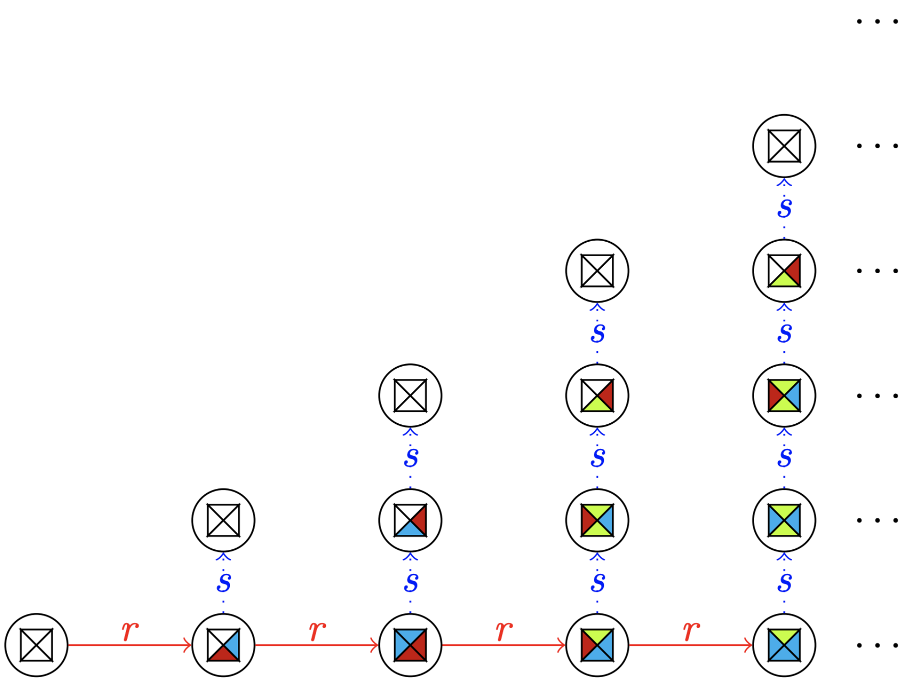
Output: Can we cover an infinite triangle (a.k.a. octant) according to tiling rules?



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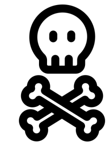
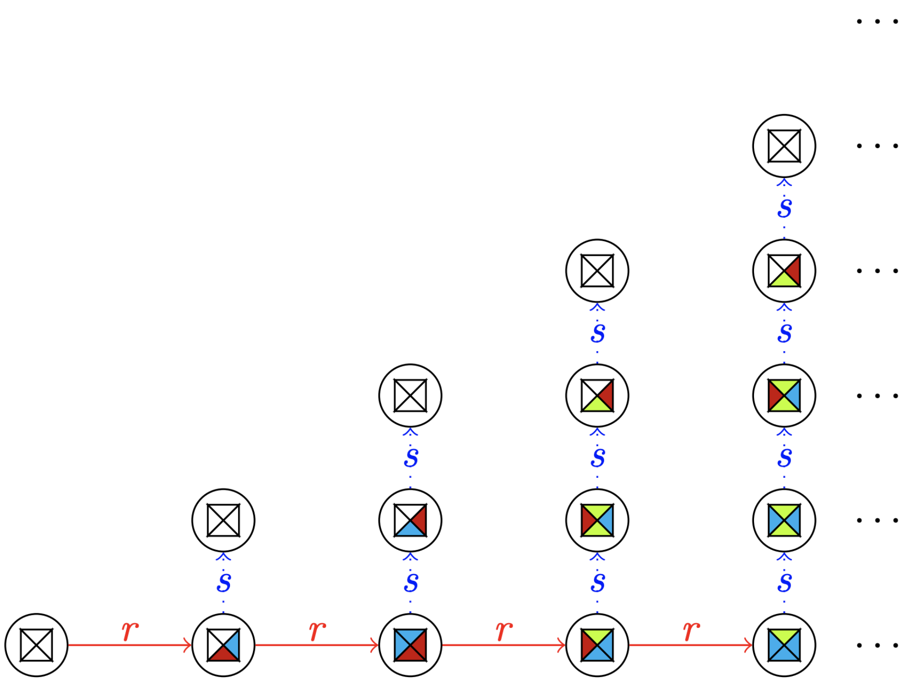


Proof idea: the ontology defines "octant-like" models and the query detects errors with tiling.



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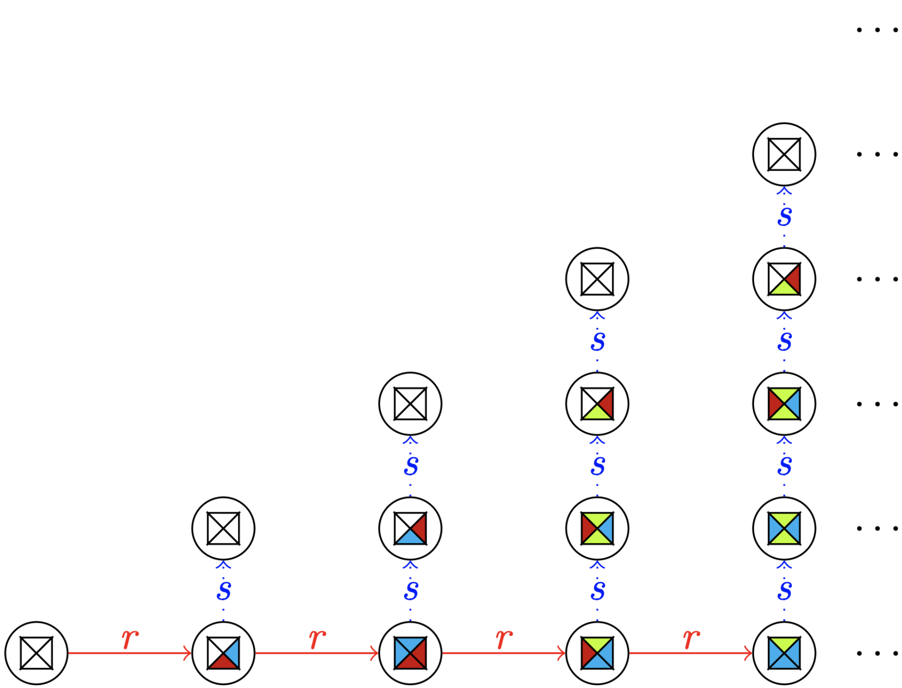

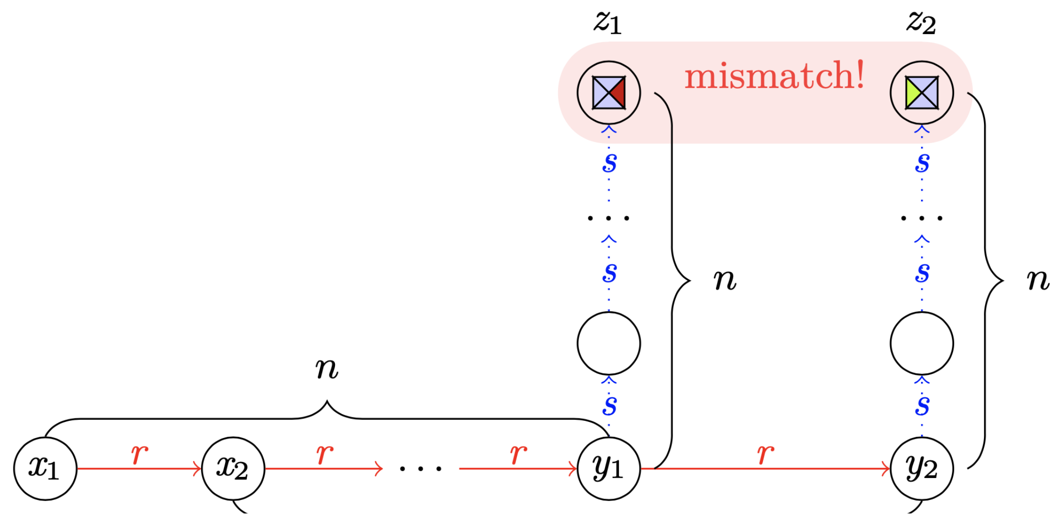

$$q^{\mathcal{D}}_{\blacktriangle} := \bigvee_{\mathrm{t},\mathrm{t}' \text{ violating (OHori)}} \big[r(x_1,x_2) \wedge r^*(x_2,y_1) \wedge r(y_1,y_2) \wedge s^*(y_1,z_1) \wedge s^*(y_2,z_2)$$
$$\wedge\, r^\# s^\#(x_1,z_1) \wedge r^\# s^\#(x_2,z_2) \wedge \mathrm{C}_{\mathrm{t}}(z_1) \wedge \mathrm{C}_{\mathrm{t}'}(z_2)\big]$$

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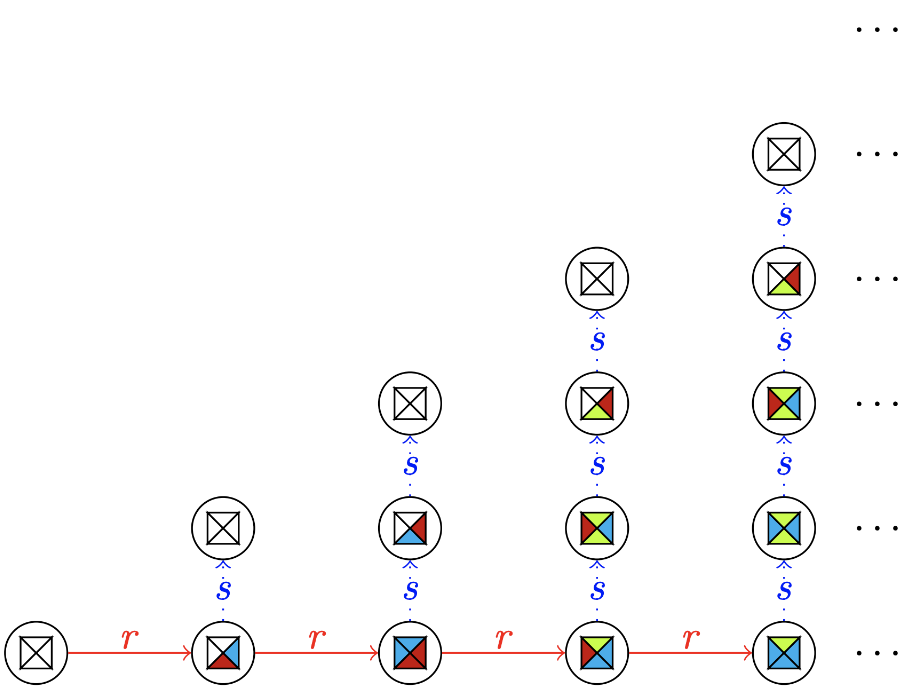

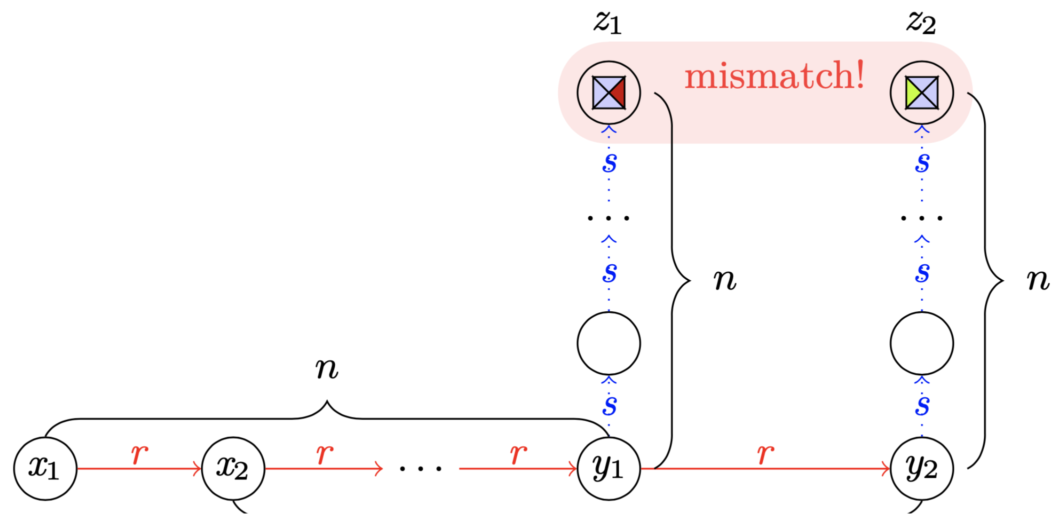

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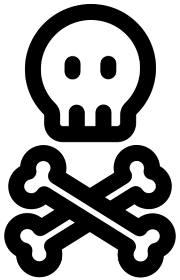
Key Property: $\mathrm{C} \not\models q$ iff the octant can be covered.

Decidability of (extensions of) $\mathcal{ALC}_{\mathsf{reg}}$ do not transfer well to the non-regular setting.

Decidability of (extensions of) $\mathcal{ALC}_{\mathsf{reg}}$ do not transfer well to the non-regular setting.

Loops



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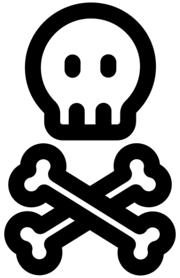
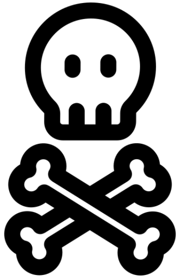
Loops



Nominals

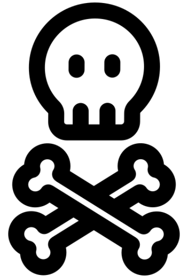
Decidability of (extensions of) $\mathcal{ALC}_{\mathsf{reg}}$ do not transfer well to the non-regular setting.

Loops



Nominals

Queries



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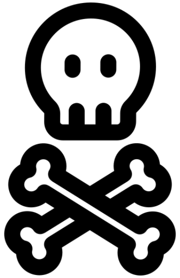
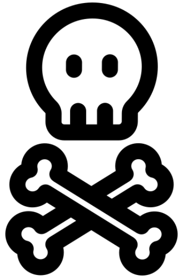
Loops

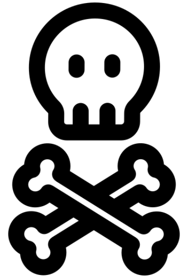


Vis. 1-counter

Nominals

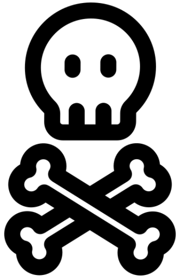


Queries



Decidability of (extensions of) $\mathcal{ALC}_{\mathsf{reg}}$ do not transfer well to the non-regular setting.

Loops

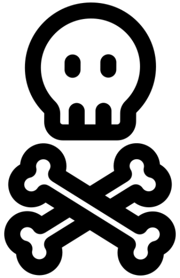


Vis. 1-counter

Nominals

$\mathcal{ALC}_{\mathsf{reg}}^{r\#s\#}$

Queries



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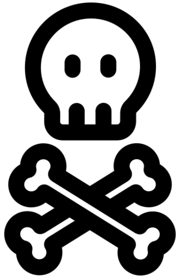
Loops



Vis. 1-counter

Nominals

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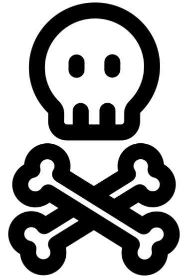
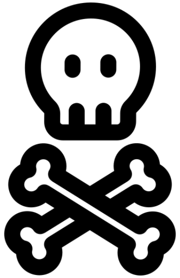
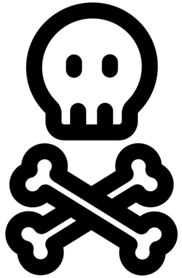
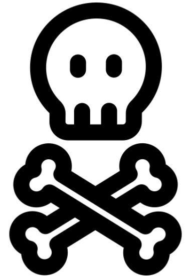
Queries



$\mathcal{ALC}$ + CRPQs with $r^{\#}s^{\#}$

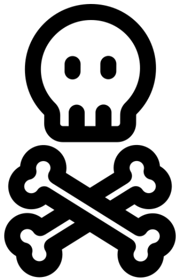
Decidability of (extensions of) $\mathcal{ALC}_{\text{reg}}$ do not transfer well to the non-regular setting.

Loops	**Nominals**	**Queries**
		
Vis. 1-counter	$\mathcal{ALC}_{\text{reg}}^{r^{\#}s^{\#}}$	$\mathcal{ALC}$ + CRPQs with $r^{\#}s^{\#}$

Open Problem 1: Incorporating ABoxes?

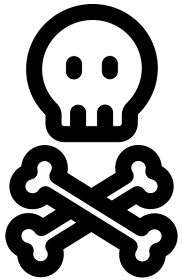
Decidability of (extensions of) $\mathcal{ALC}_{\mathsf{reg}}$ do not transfer well to the non-regular setting.

Loops



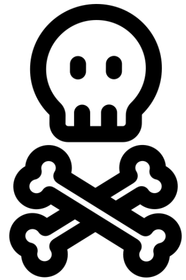
Vis. 1-counter

Nominals



$\mathcal{ALC}_{\mathsf{reg}}^{r^{\#}s^{\#}}$

Queries

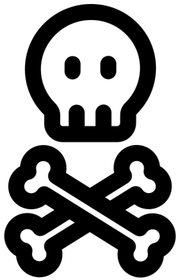
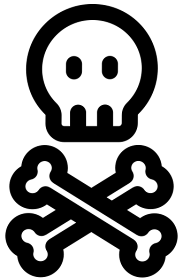
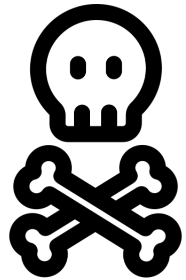


$\mathcal{ALC}$ + CRPQs with $r^{\#}s^{\#}$

Open Problem 1: Incorporating ABoxes?

Open Problem 2: Finite Satisfiability of $\mathcal{ALC}_{\mathsf{vpl}}$?

Decidability of (extensions of) $\mathcal{ALC}_{\mathbf{reg}}$ do not transfer well to the non-regular setting.

Loops	**Nominals**	**Queries**
		
Vis. 1-counter	$\mathcal{ALC}_{\mathrm{reg}}^{r\#s\#}$	$\mathcal{ALC}$ + CRPQs with $r^{\#}s^{\#}$

Open Problem 1: Incorporating ABoxes?

Open Problem 2: Finite Satisfiability of $\mathcal{ALC}_{\mathrm{vpl}}$?

Open Problem 3: Sharpen undecidability for $\mathcal{ALC}_{\mathrm{vpl}}$ with Self?

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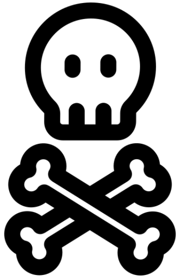
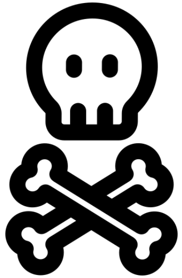
Loops



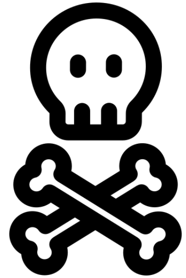
Vis. 1-counter

Nominals



$\mathcal{ALC}^{r^{\#}s^{\#}}_{\mathsf{reg}}$

Queries



$\mathcal{ALC}$ + CRPQs with $r^{\#}s^{\#}$

Looking for (postdoc?) job from Sept'24!

See: bartoszjanbednarczyk.github.io

Open Problem 1: Incorporating ABoxes?

Open Problem 2: Finite Satisfiability of $\mathcal{ALC}_{\mathsf{vpl}}$?

Open Problem 3: Sharpen undecidability for $\mathcal{ALC}_{\mathsf{vpl}}$ with Self?