Worst-Case Optimal Querying of Very Expressive Description Logics with Path Expressions and Succinct Counting

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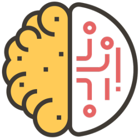
Running example ($\mathcal{ZOIQ}$ KB)

Database

HasParent (Heracles, Zeus)
HasParent (Perseus, Zeus)
male (Zeus)
deity (Zeus)
mortal (Alcmene)

Knowledge



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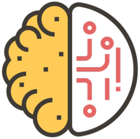

$$
\begin{array}{rcl}
\mathit{mortal} & \sqsubseteq & \neg \mathit{deity} \\
\top & \sqsubseteq & \exists \mathit{HasFather}.\mathit{male} \sqcap \exists \mathit{HasMother}.\mathit{female} \\
\mathit{HasParent} & \equiv & \mathit{HasMother} \cup \mathit{HasFather} \\
\forall \mathit{HasParent}.\mathit{mortal} & \sqsubseteq & \mathit{mortal} \\
\mathit{deity} & \sqsubseteq & \forall \mathit{HasParent}^{*}.\mathit{deity}
\end{array}
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Positive 2-Way Regular Path Query

$$\exists x, y, z \; \underbrace{(\mathrm{HasParent}^{*} \circ \mathrm{HasParent}^{-^{*}})(x, y) \wedge \mathrm{HasParent}(z, x) \wedge \mathrm{HasParent}(z, y)}_{\text{x and y are relatives with a common children z}}$$

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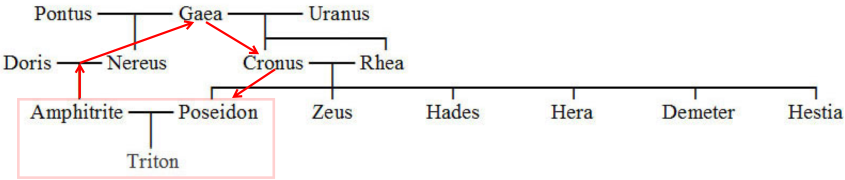
An example match π in a model $\mathcal{I}$:

$\pi(x) = \mathrm{Amphitrite}^{\mathcal{I}}$ $\pi(y) = \mathrm{Poseidon}^{\mathcal{I}}$ $\pi(z) = \mathrm{Triton}^{\mathcal{I}}$



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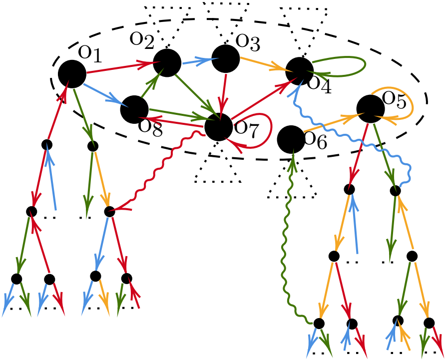
Extensions of $\mathcal{Z}$

Even more expressive logics: $\mathcal{ZIQ}$, $\mathcal{ZOQ}$ and $\mathcal{ZOI}$

Quasi-forest model property (QFMP)



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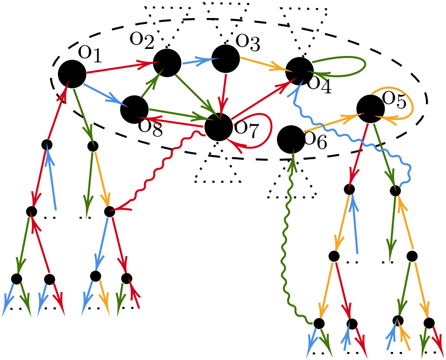

QFMP for $\mathcal{Z}$ family [Calvanese et al, IJCAI'09]

Logics $\mathcal{ZIQ}$, $\mathcal{ZOQ}$ and $\mathcal{ZOI}$ enjoy QFMP.
SAT "only" ExpTime-complete (with bin encoding of $\mathcal{Q}$).

Querying $\mathcal{Z}$ with P2RPQs (existing results)

P2RPQ entailment for $\mathcal{Z}$ family [Calvanese et al, IJCAI'09]

Testing P2RPQ entailment for $\mathcal{ZIQ}, \mathcal{ZOQ}, \mathcal{ZOI}$ can be done in 3ExpTime (2ExpTime-c. under unary encoding).

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- Quite complicated...
- Heavy machinery on automata theory...

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- Reduction to satisfiability (works under binary enc)
- P2RPQ version of so-called "rolling-up" technique used for CQs
- Simulate automata on quasi-forest-models = Match calculus

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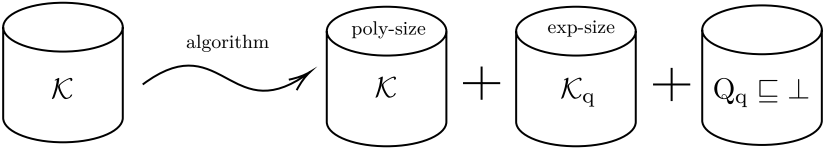
  - basically we simulate automaton on quasi-forest models.



Generated with match calculus

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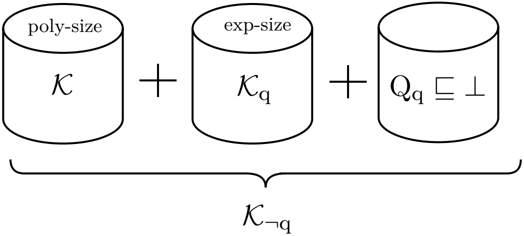

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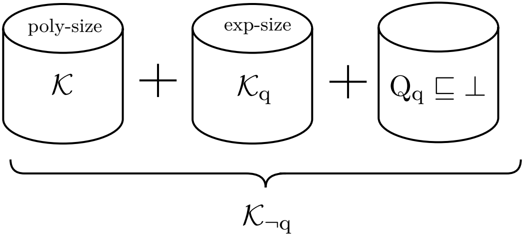

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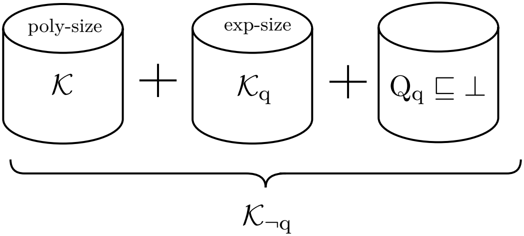

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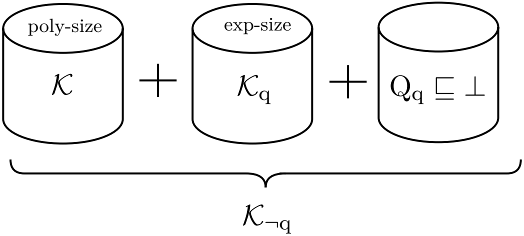

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- Thus in 2ExpTime w.r.t. $|\mathcal{K}| + |\mathrm{q}|$

Applications to other logics

We presented a reduction form $\mathcal{GC}^2$ to $\mathcal{ZIQ}$, hence we conclude:

Positive regular path queries in $\mathcal{GC}^2$

P2RPQ entailment for $\mathcal{GC}^2$ is 2ExpTime-complete.

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By reusing exponential reduction from $\mathcal{SR}$ to $\mathcal{Z}$:

Positive regular path queries in $\mathcal{SR}$ family

P2RPQ entailment for $\mathcal{SR}(\mathcal{OI}, \mathcal{IQ}, \mathcal{OQ})$ is in 3ExpTime.

Applications to query containment

Query containment

Testing query containment $\mathcal{K} \models \mathrm{q} \subseteq \mathrm{q}'$ is:
in 2ExpTime for:

- $\mathcal{K}$ in $\mathcal{ZOQ}$ or $\mathcal{ZOI}$ and $\mathrm{q}, \mathrm{q}' \in$ P2RPQ
- $\mathcal{K}$ in $\mathcal{ZIQ}$ and $\mathrm{q} \in$ P2RPQ , $\mathrm{q}' \in$ CQ

and in 3ExpTime for:

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Moreover once the number of the atoms from the query is bounded complexities of each problem drops by one exponential.

Conclusions and open problems

Our results

P2RPQ entailment for $\mathcal{ZIQ}, \mathcal{ZOQ}, \mathcal{ZOI}$ is 2ExpTime-c +
P2RPQ entailment for $\mathcal{SRIQ}, \mathcal{SROQ}, \mathcal{SROI}$ in 3Exp +
P2RPQ containment in 2ExpTime +
One exp less for all problems when #atoms(q) $\leq$ Const.

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- Data complexity?
- Finite query entailment?
- Sat of $\mathcal{ZOIQ}$?

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