A Framework for Reasoning about Dynamic Axioms

in Description Logics

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TU Dresden & University of Wrocław, CNRS & ENS Paris-Saclay

TECHNISCHE UNIVERSITÄT DRESDEN

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Running example: basketball teams and (possibly injured) players

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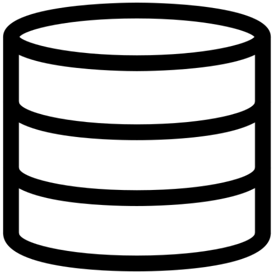

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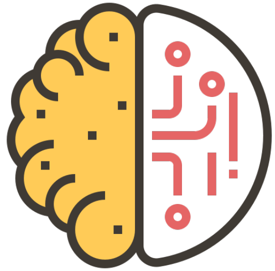

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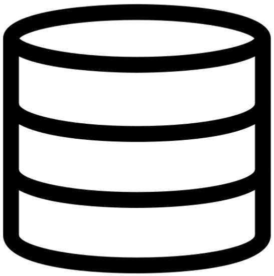

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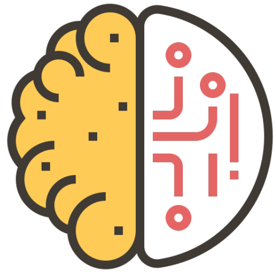

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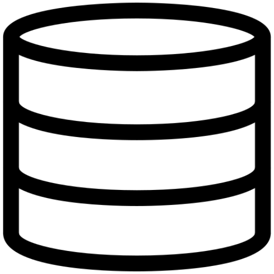

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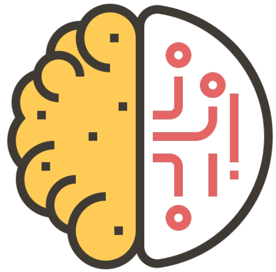

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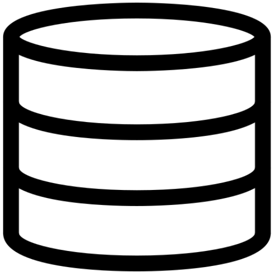

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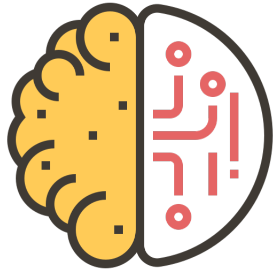

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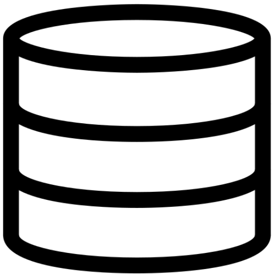

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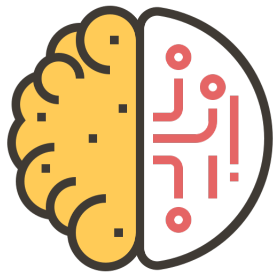

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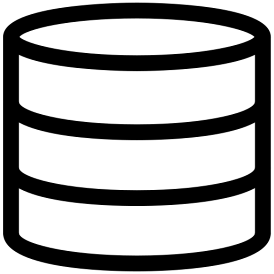

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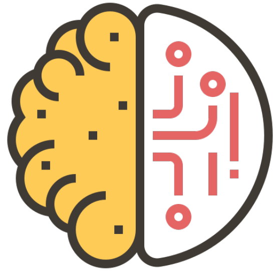

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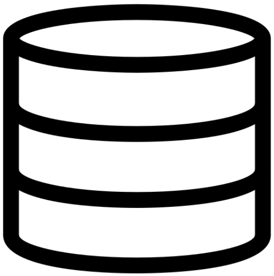

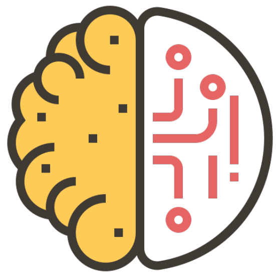

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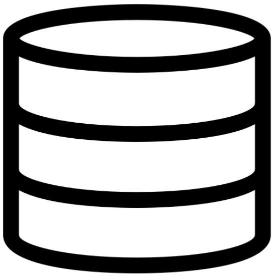

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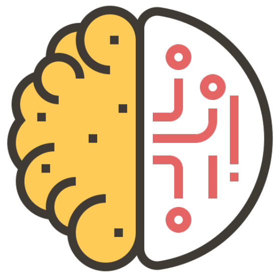

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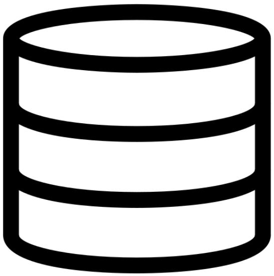

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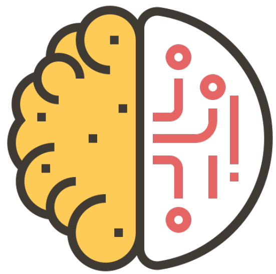

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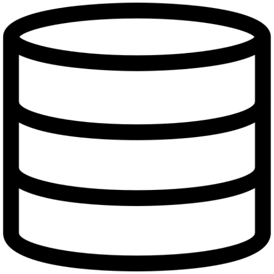

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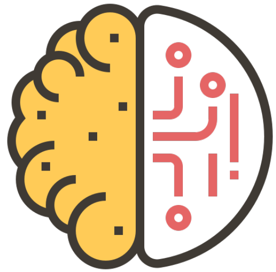

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We propose a new framework based on separation logics!

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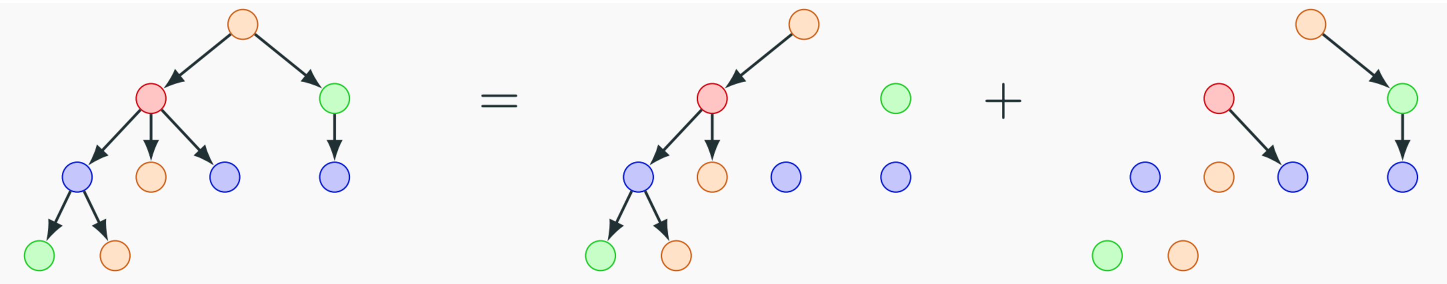

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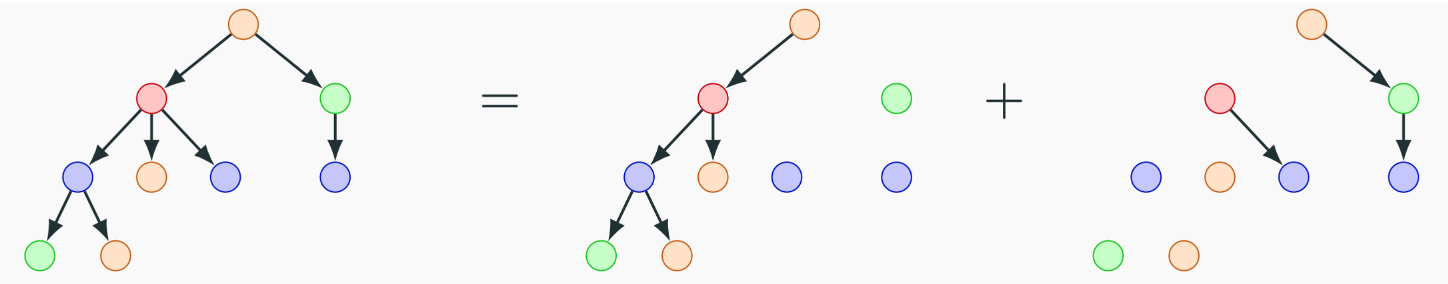

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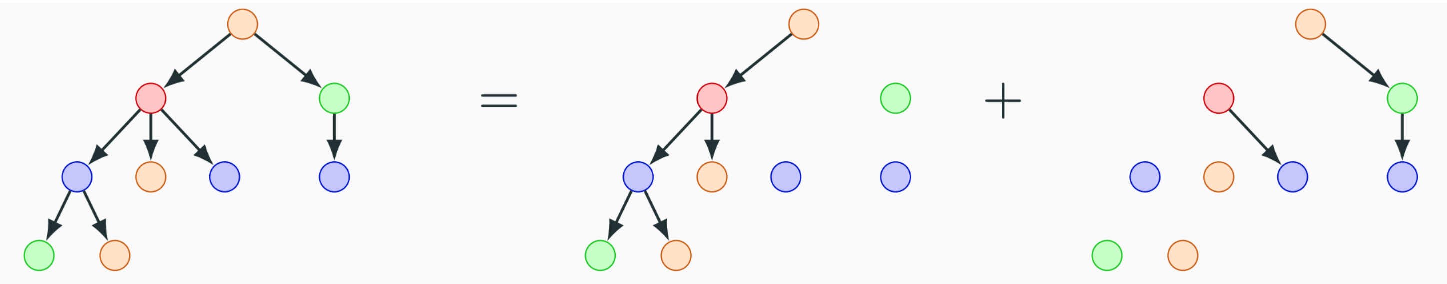

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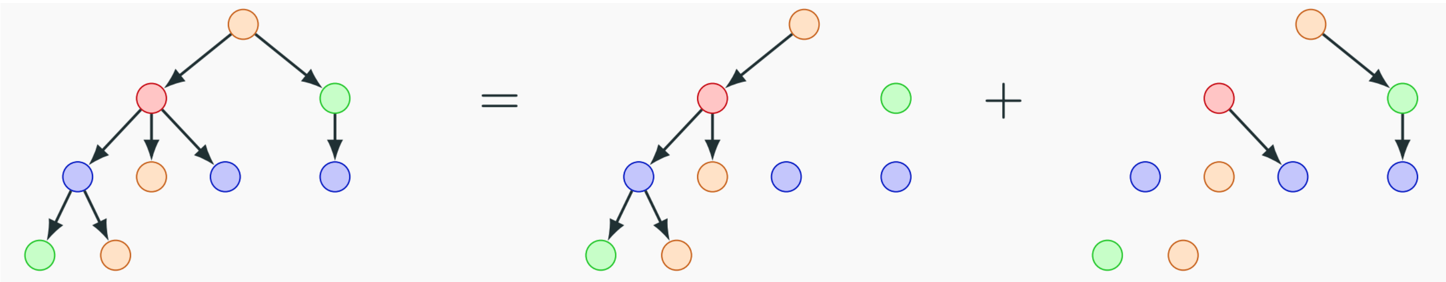

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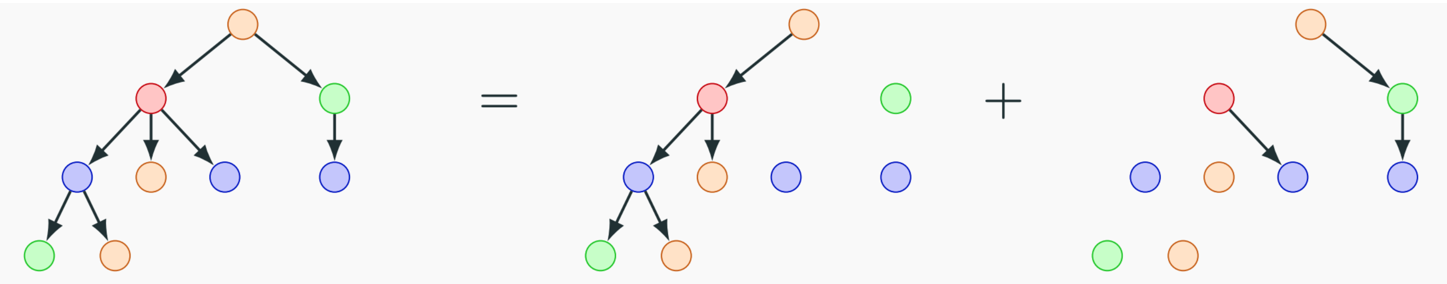

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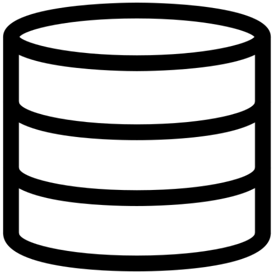

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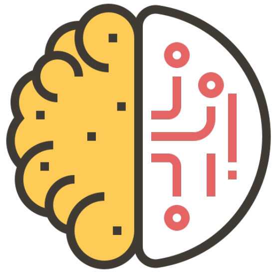

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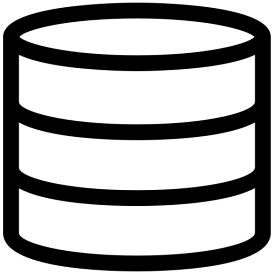

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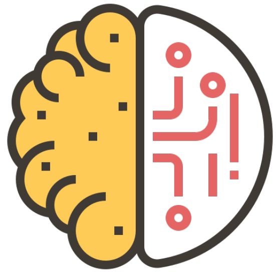

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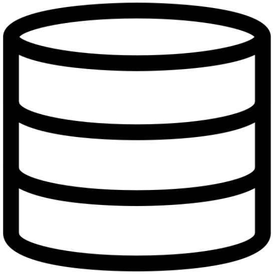

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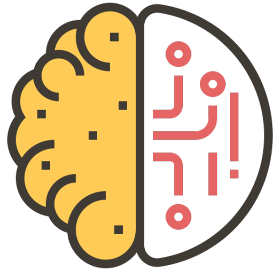

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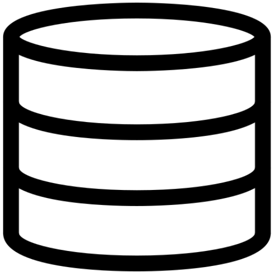

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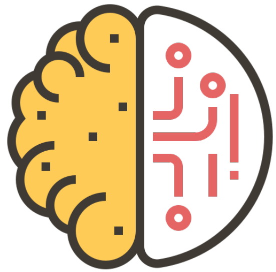

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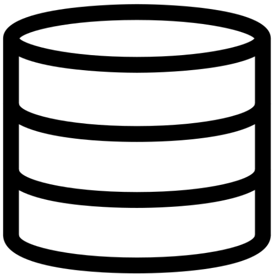

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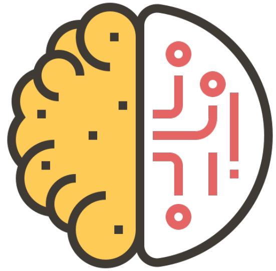

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$\mathcal{K} \cup \mathbb{U}$ is satisfiable iff there is an evolution where $\texttt{Zion}$ is drafted by $\texttt{Pelicans}$.

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Check the paper for more details!