Finite Entailment of Local Queries in the $\mathcal{Z}$ family of Description Logics

19th of September 2021, DL Workshop 2021

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TU Dresden & University of Wrocław

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Running example: Greek mythology $\mathcal{ZOIQ}$ knowledge base

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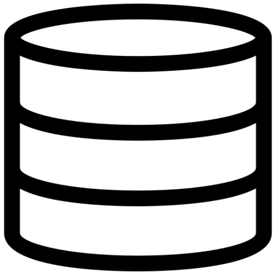
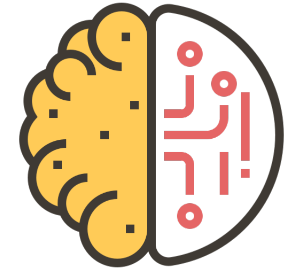


Knowledge (TBox)



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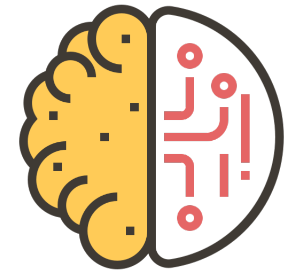

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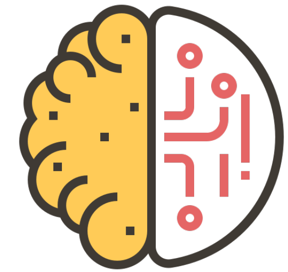

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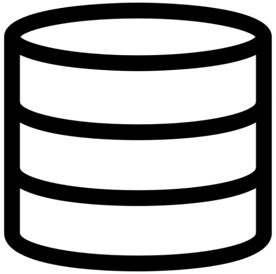

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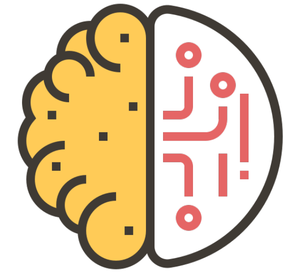

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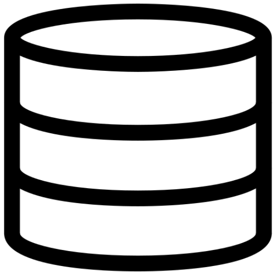

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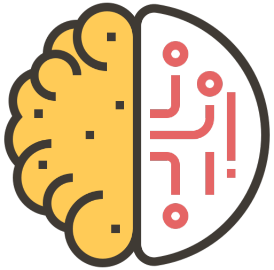

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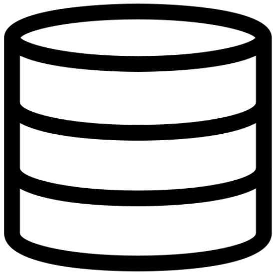

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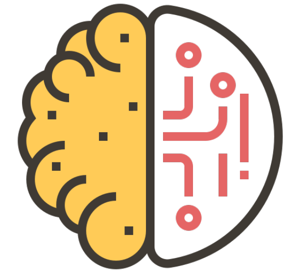

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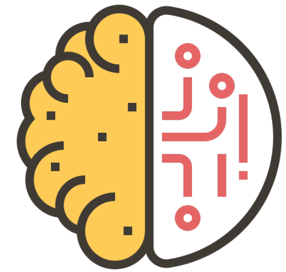

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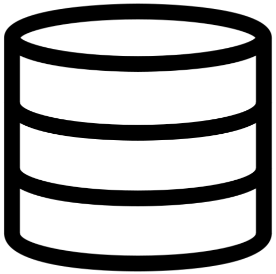

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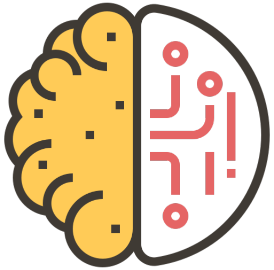

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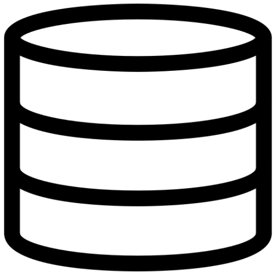

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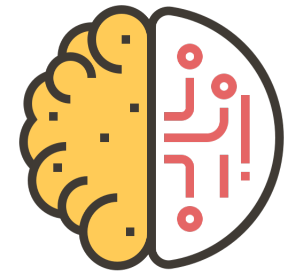

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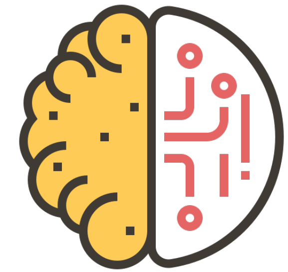

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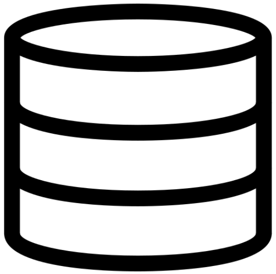

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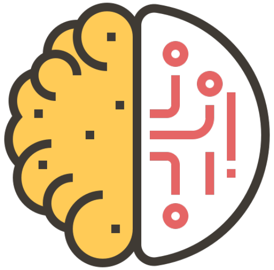

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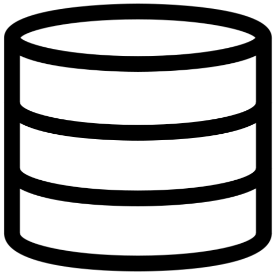

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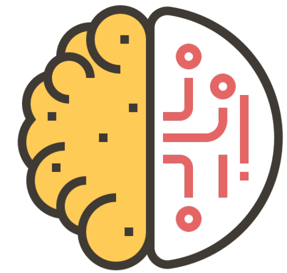

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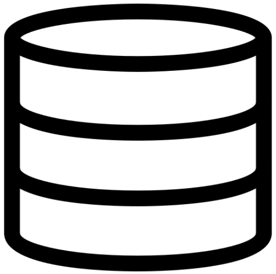

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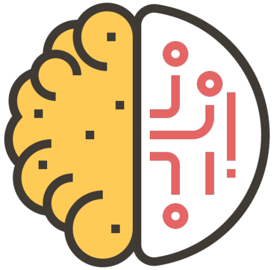

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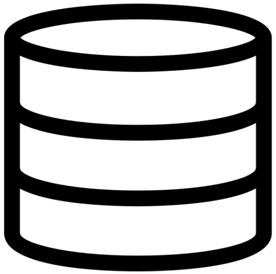

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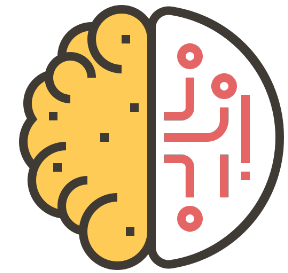

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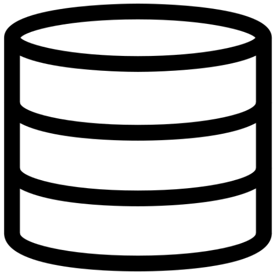

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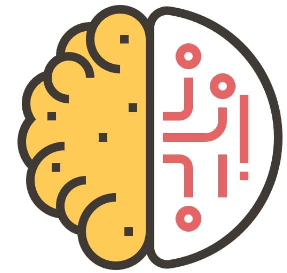

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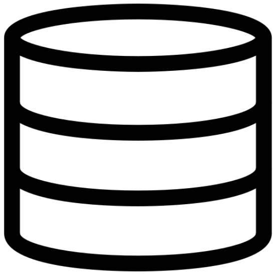

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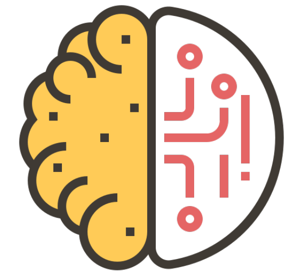

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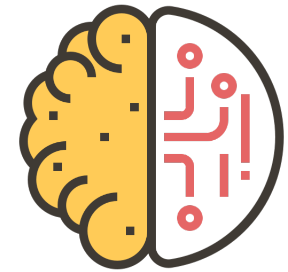

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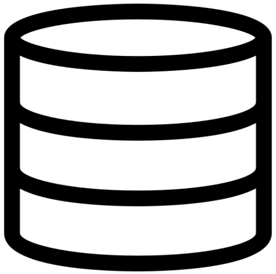

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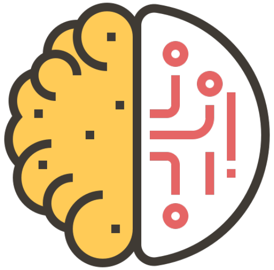

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This work: study of KBSat and CQ Entailment in the finite for fragments of $\mathcal{ZOIQ}$.

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Our paper: Study of FinKBSat & Fin CQ/UCQ/PEQ Entailment for $\mathcal{Z}(\mathcal{OI}/\mathcal{OQ}/\mathcal{IQ})$.

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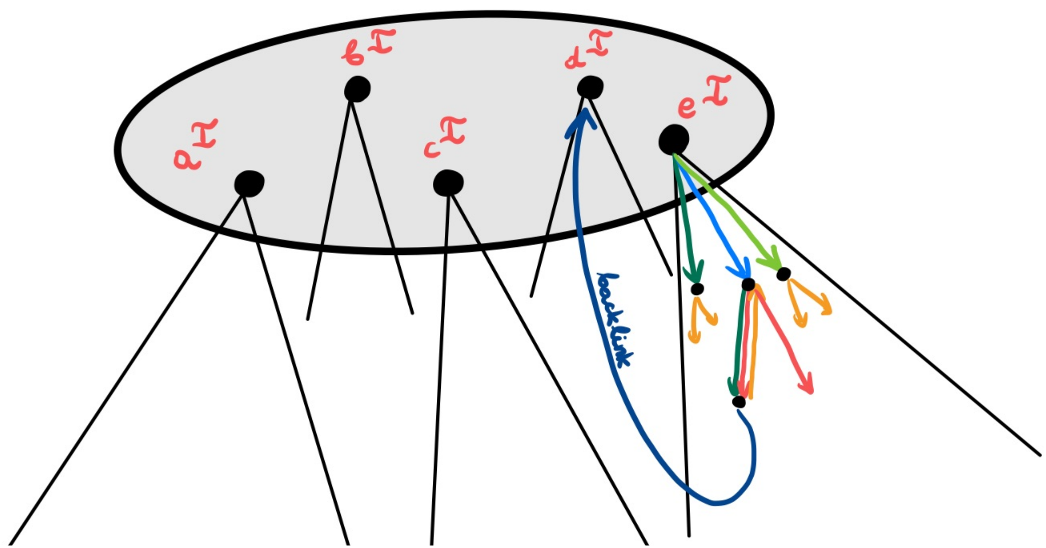

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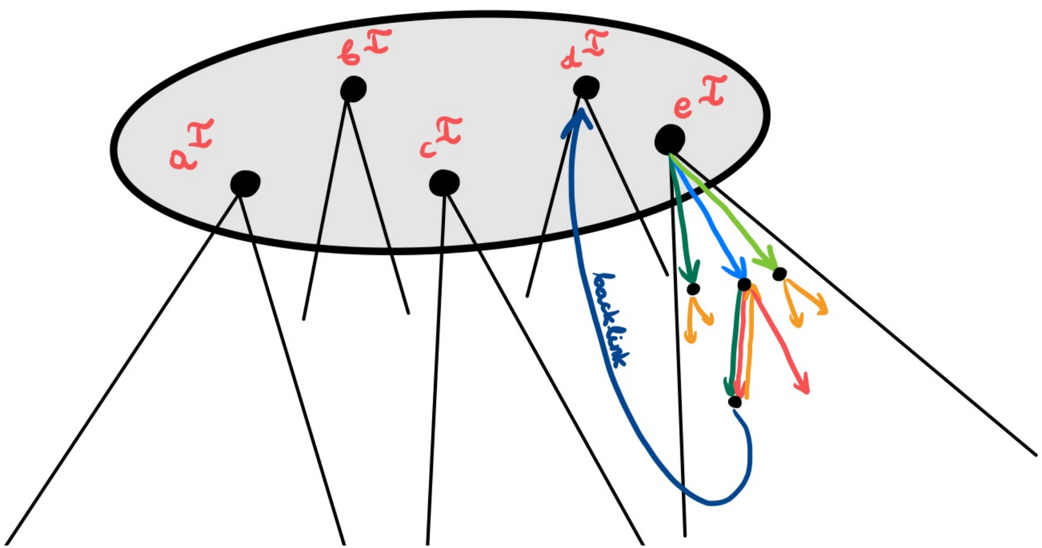

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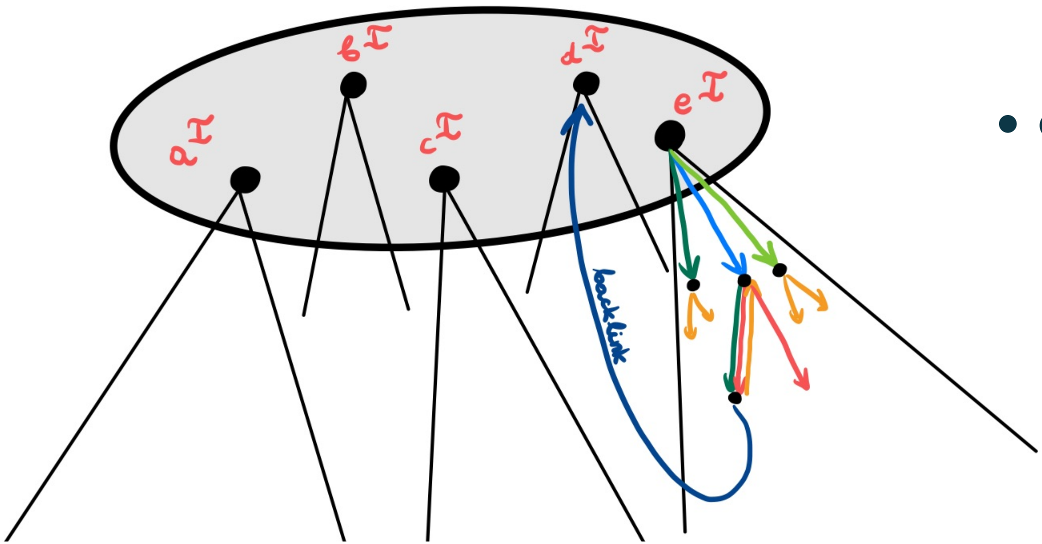

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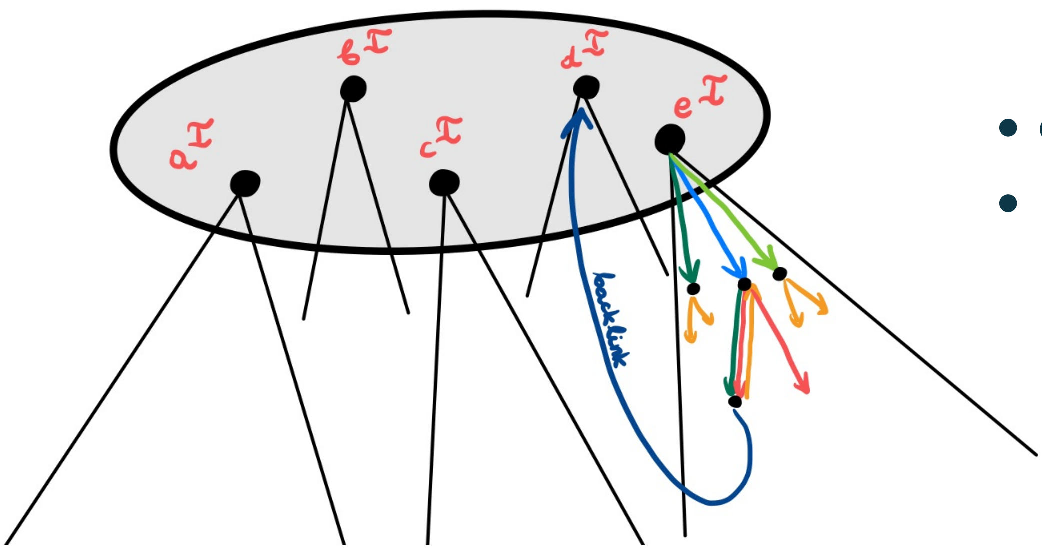

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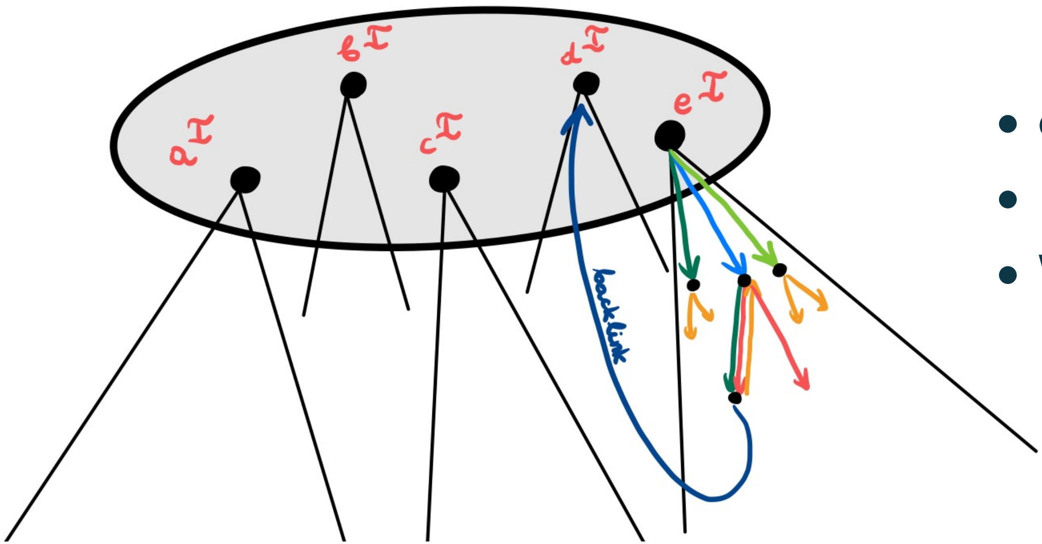

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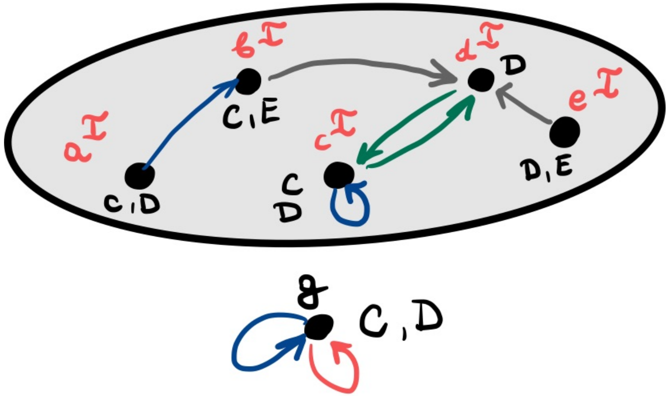

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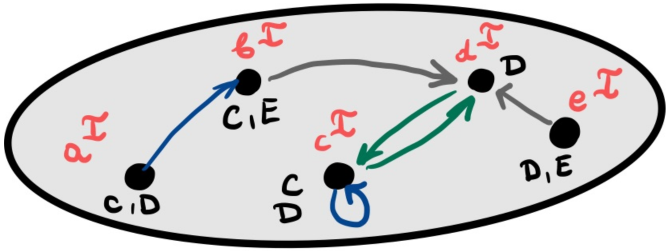

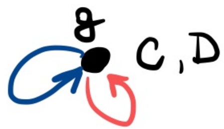

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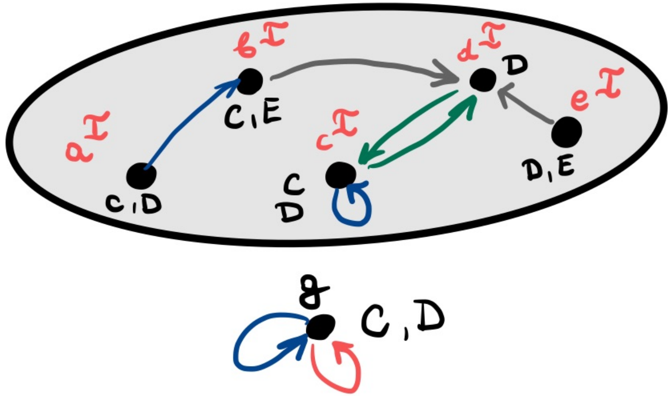

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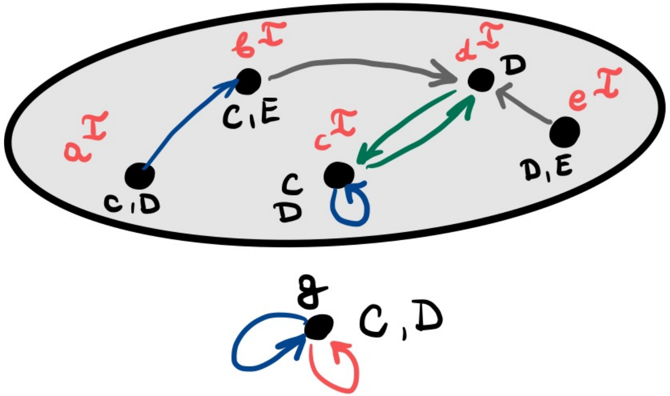

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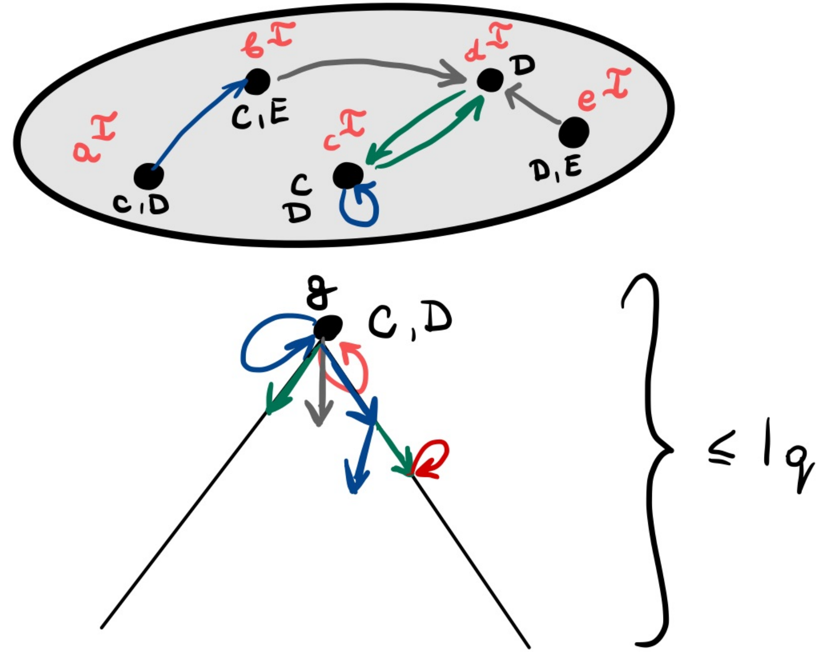

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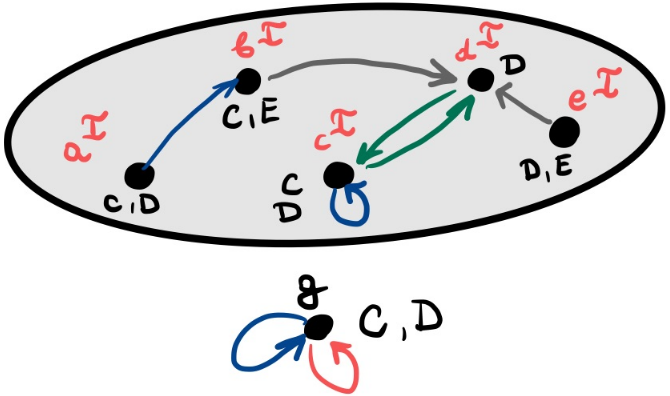

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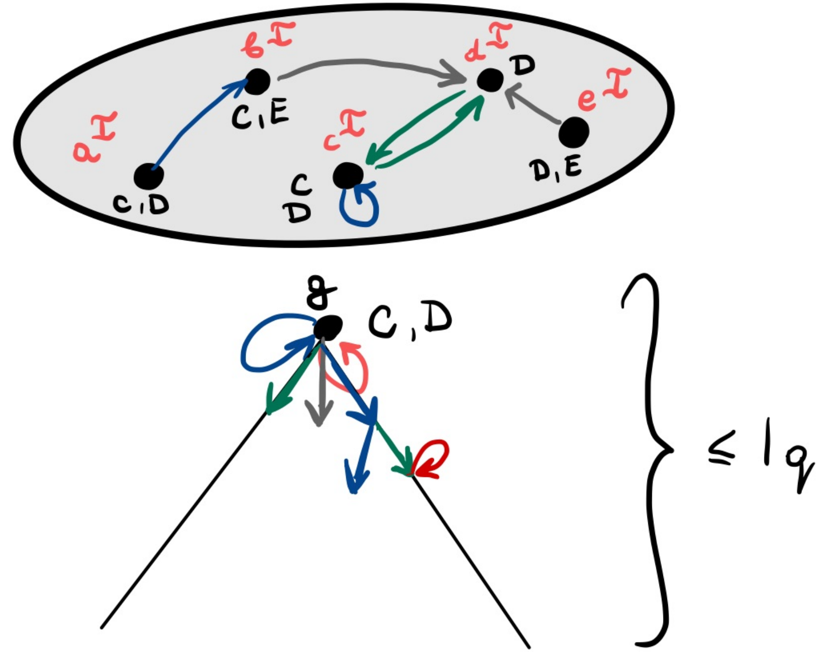

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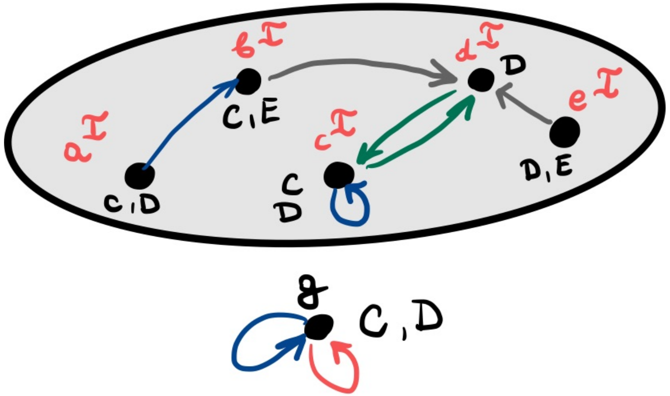
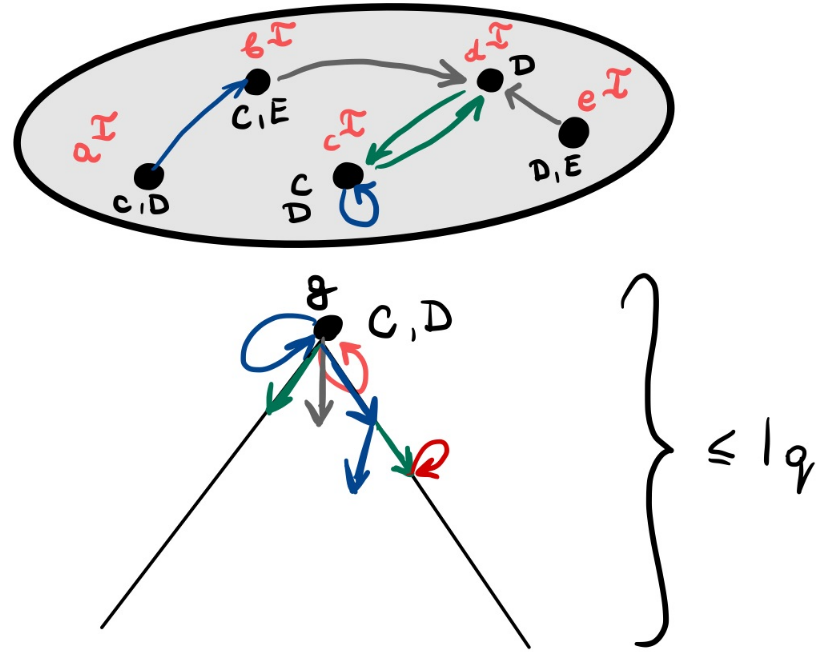


$=$ isomorphism type of $\mathcal{I}\restriction(\{\mathrm{g}\} \cup \mathsf{Nom}_{\mathcal{I}})$

Downward type of g:



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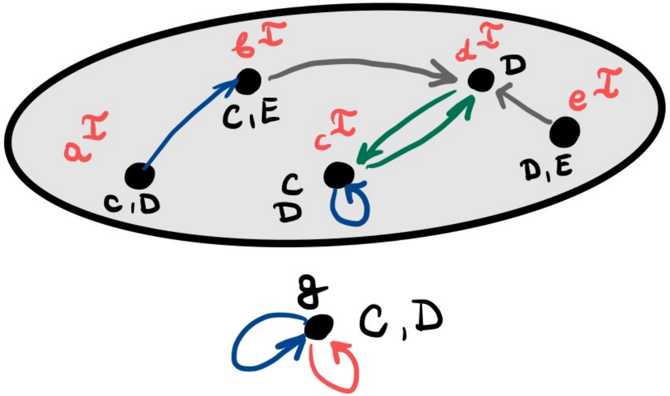

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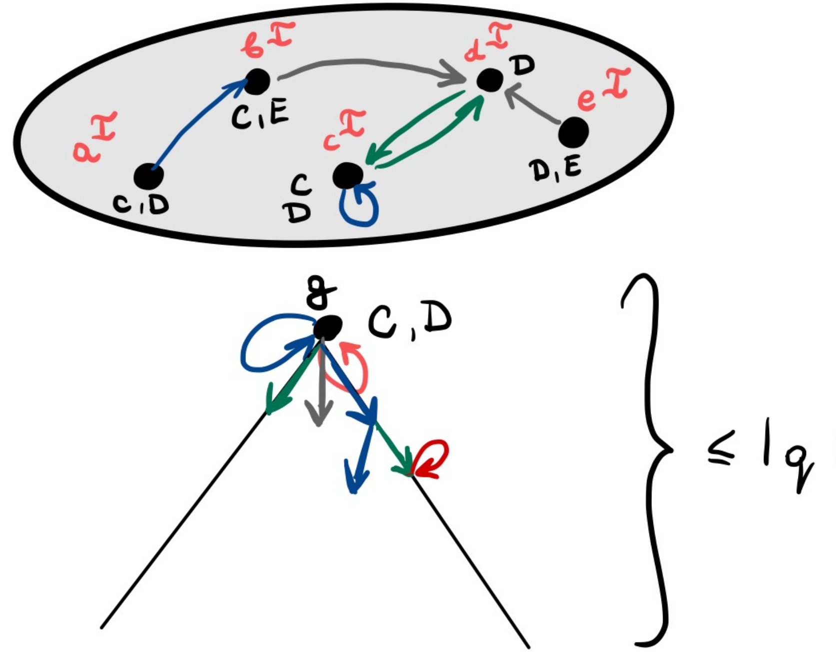

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By finite degree, we know that the set of all downward types $DTP_{\mathcal{I}}$ is finite.

We also distinguish the set of nominal downward types $NTP_{\mathcal{I}}$.

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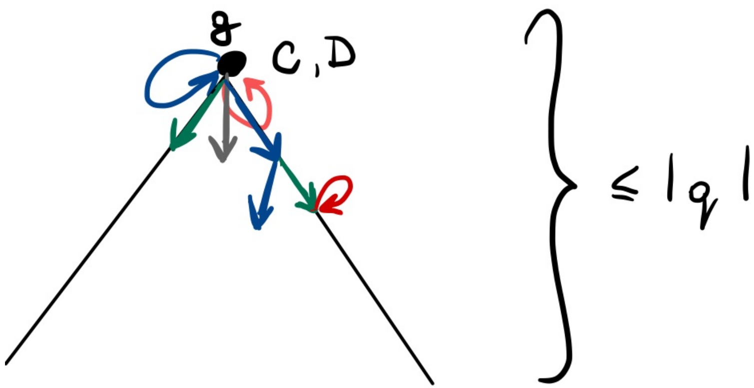

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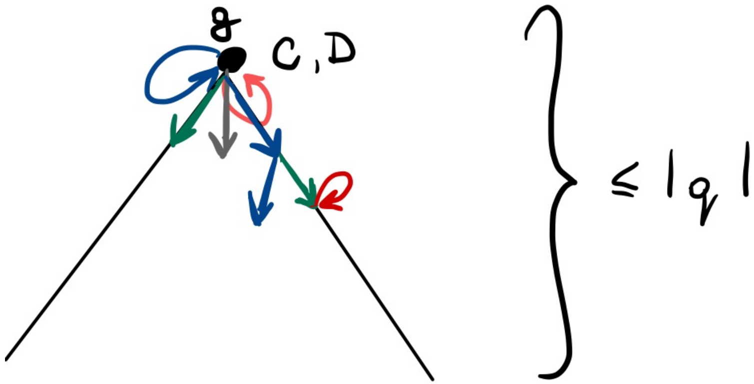

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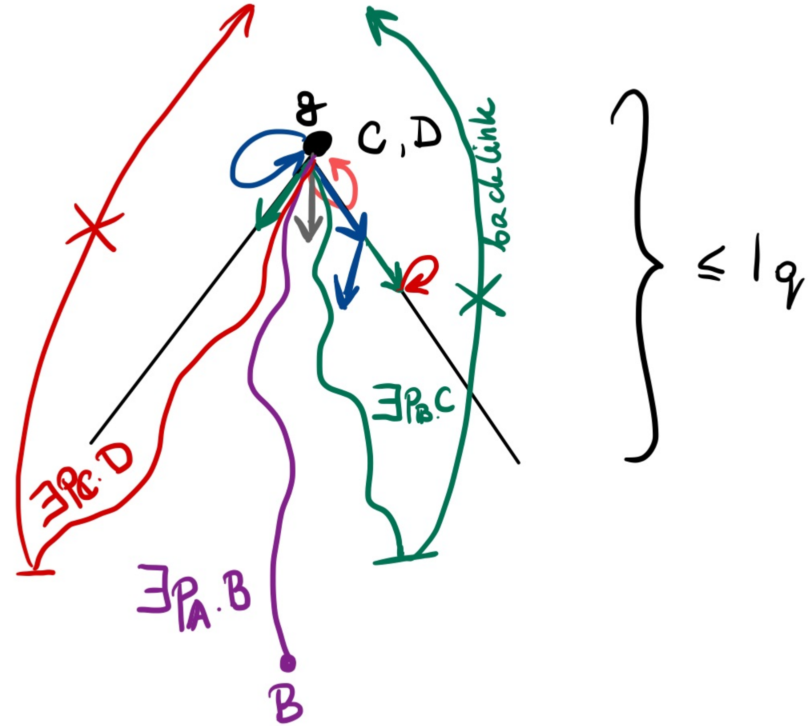

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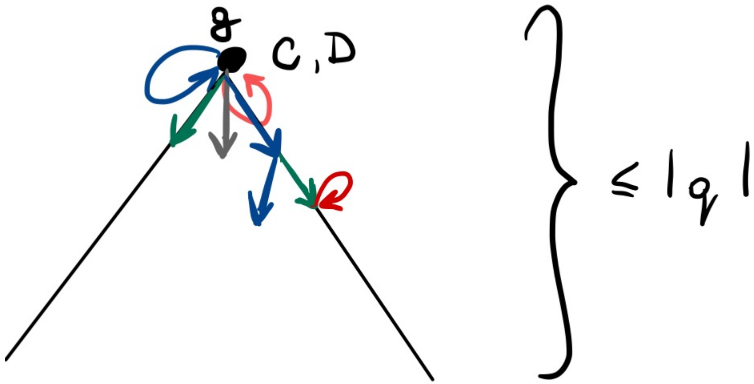

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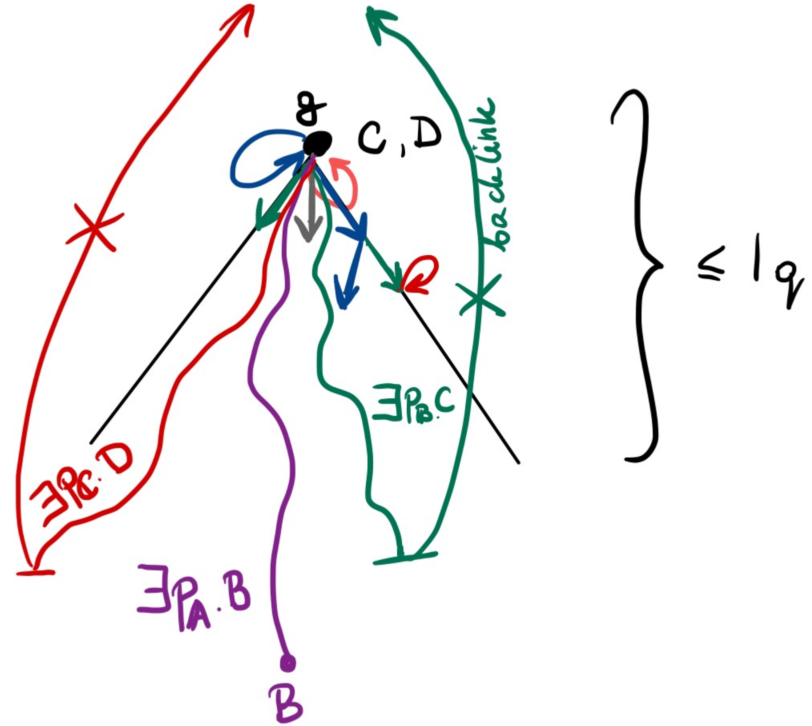

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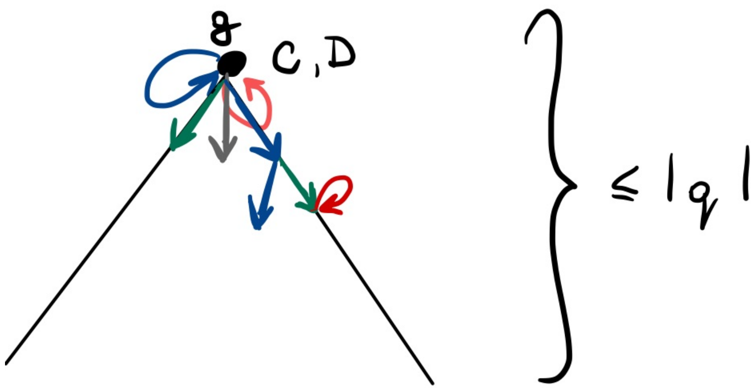

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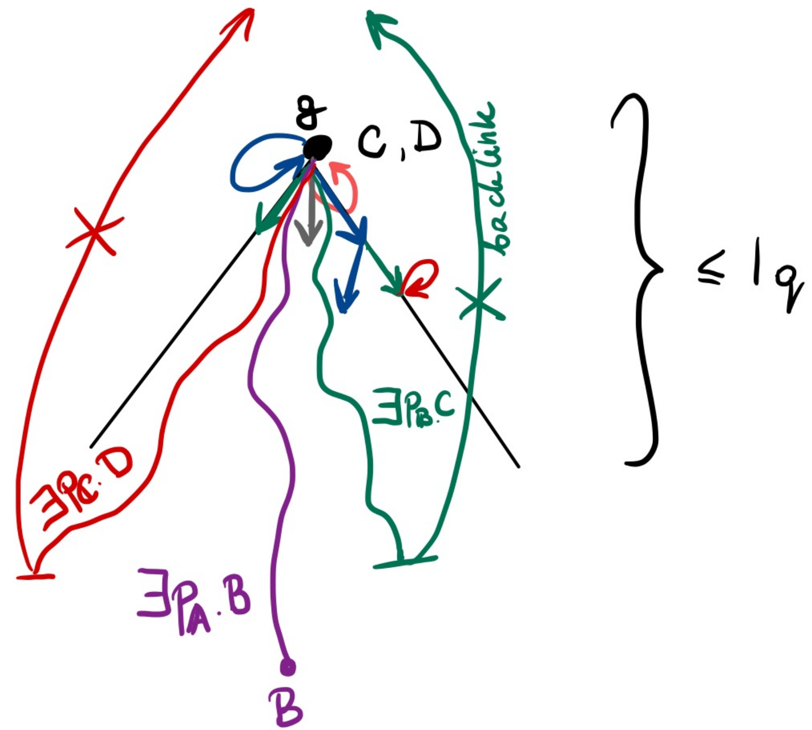

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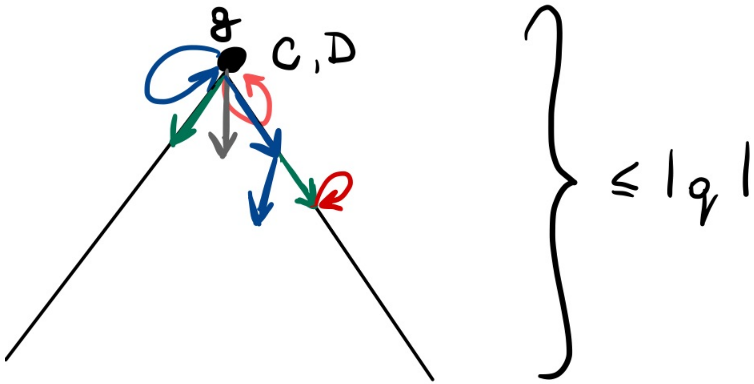

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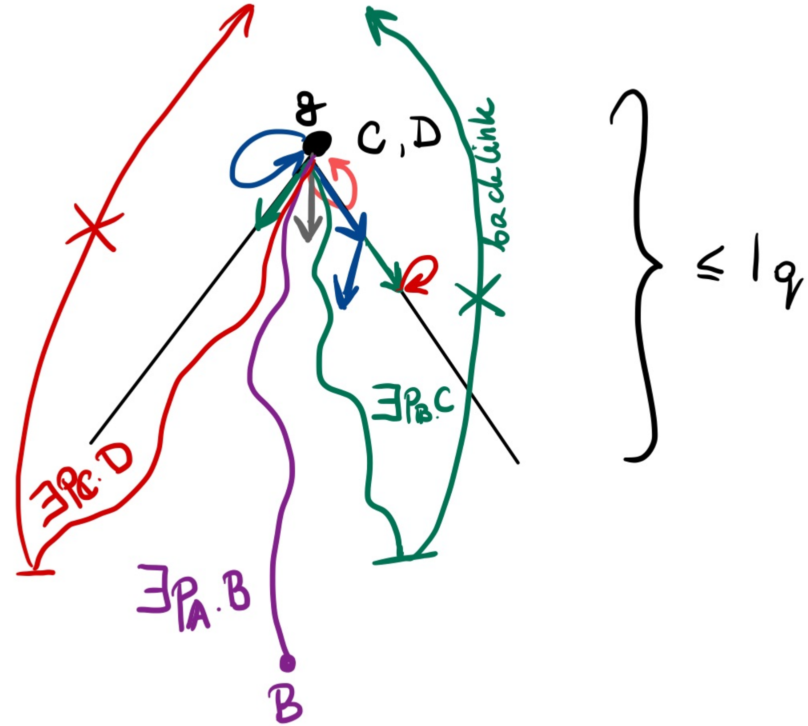

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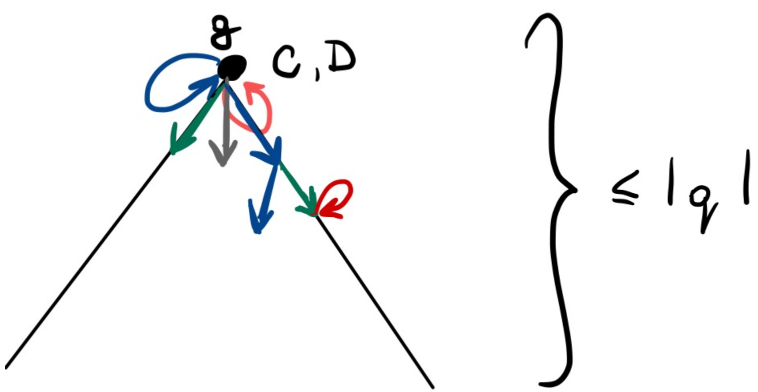

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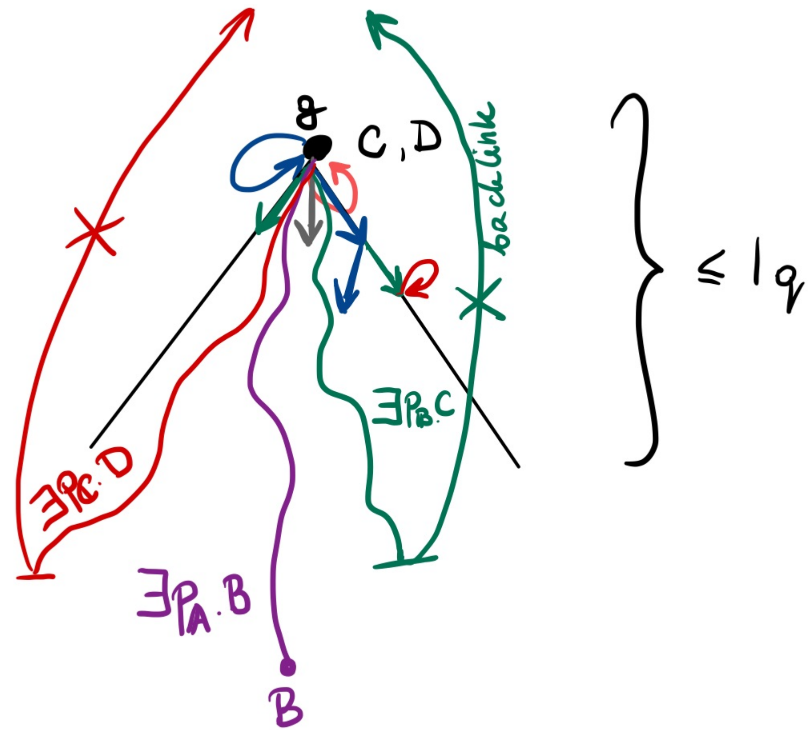

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We extend the resulting structure in a minimal way to make it parent and sibling closed.

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shade 0/1

$$\mathcal{C}^{\sigma', b}_{\sigma, \underbrace{l, m}}$$

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original type

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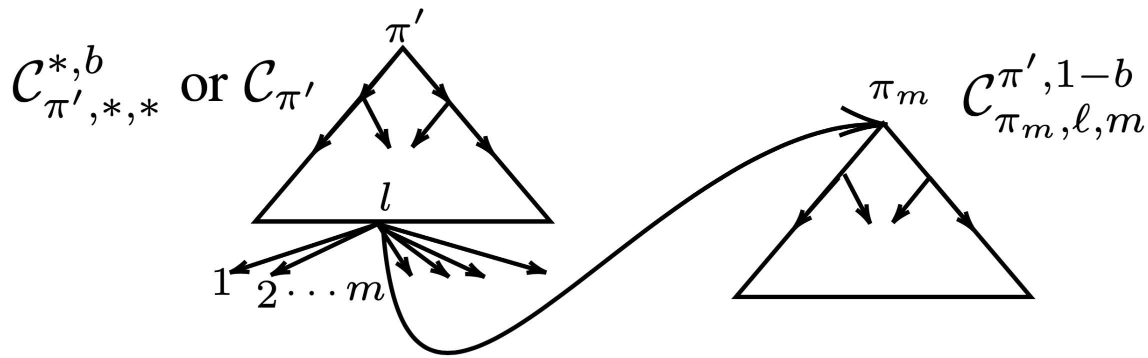

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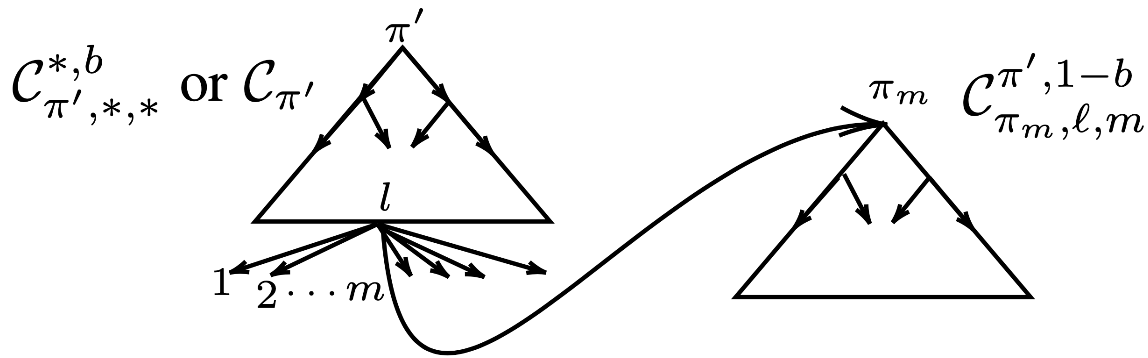

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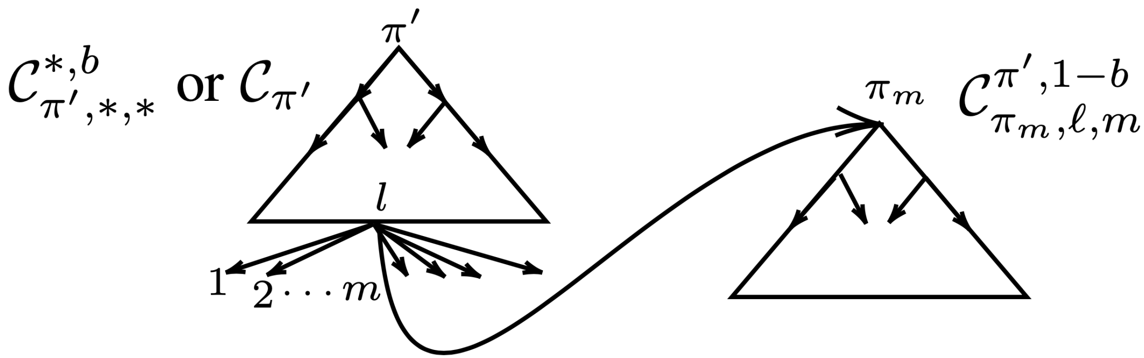

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Check Part IV: The proof in our paper! Thanks for attention!