Finite Entailment of Local Queries in the $\mathcal{Z}$ family of Description Logics

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TU Dresden & University of Wrocław

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Running example: Greek mythology $\mathcal{ZOIQ}$ knowledge base

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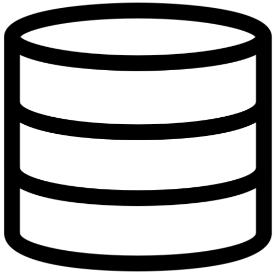

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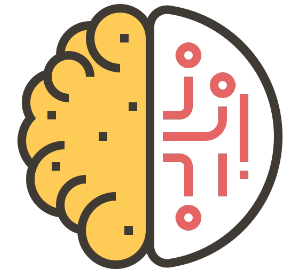

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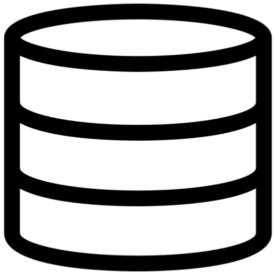

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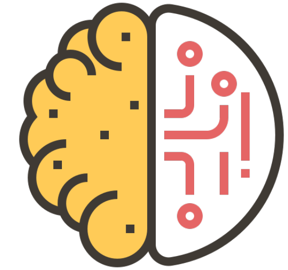

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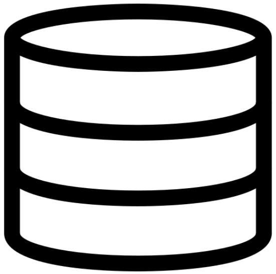

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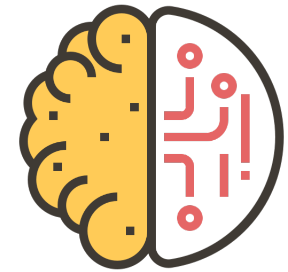

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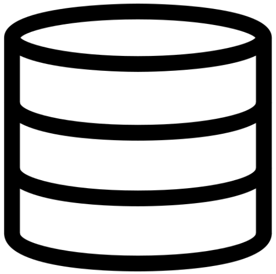

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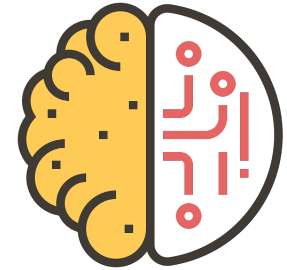

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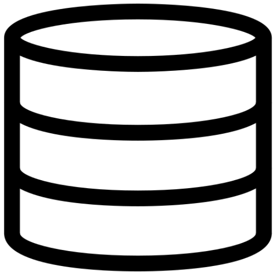

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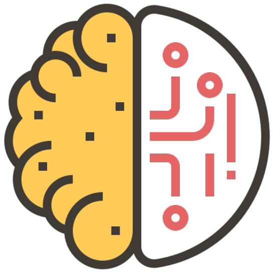

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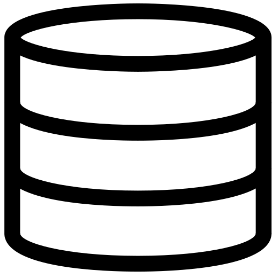

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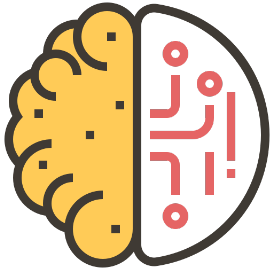

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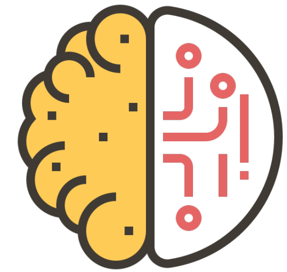

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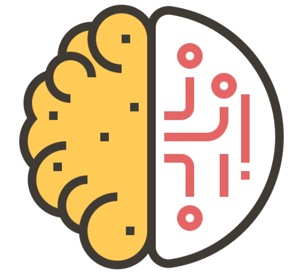

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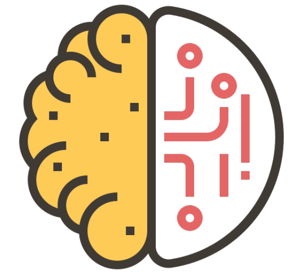

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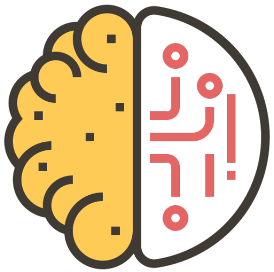

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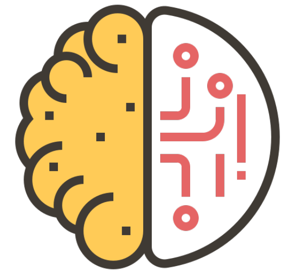

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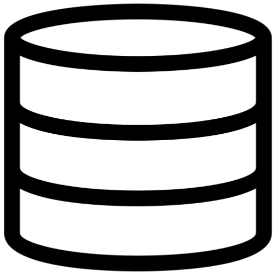

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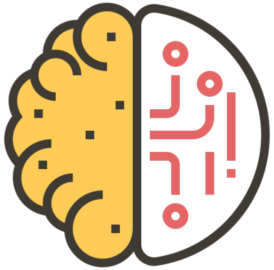

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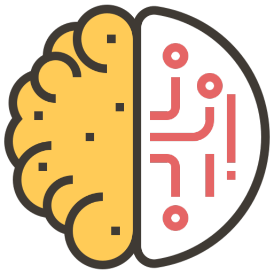

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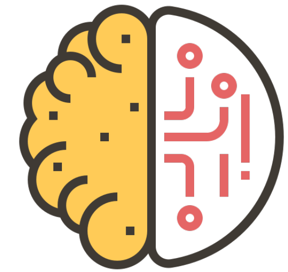

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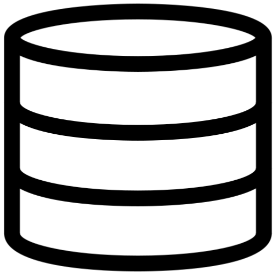

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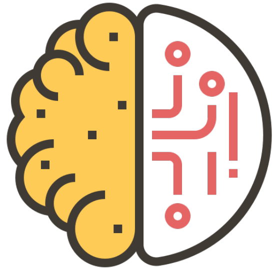

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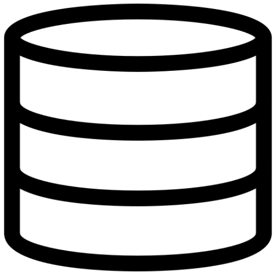

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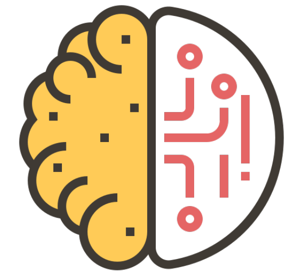

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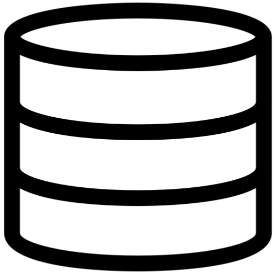

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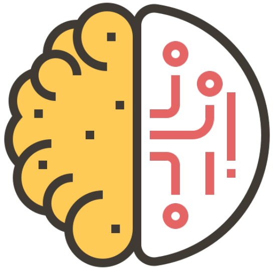

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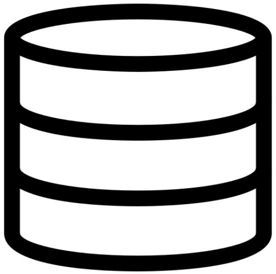

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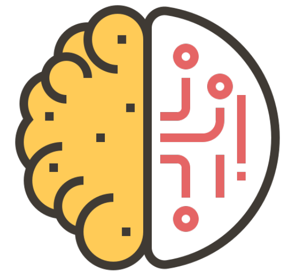

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We say that $\mathcal{L}$ is finitely-controllable (FC) iff for all $\mathcal{L}$-KBs $\mathcal{K}$ and PEQs q we have $\mathcal{K} \models q$ iff $\mathcal{K} \models_{\text{fin}} q$.

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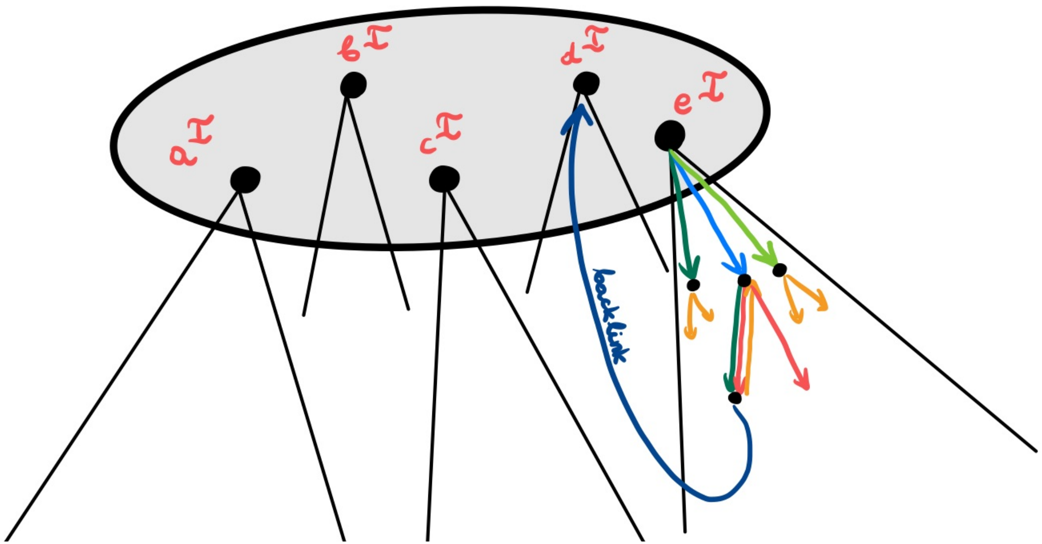

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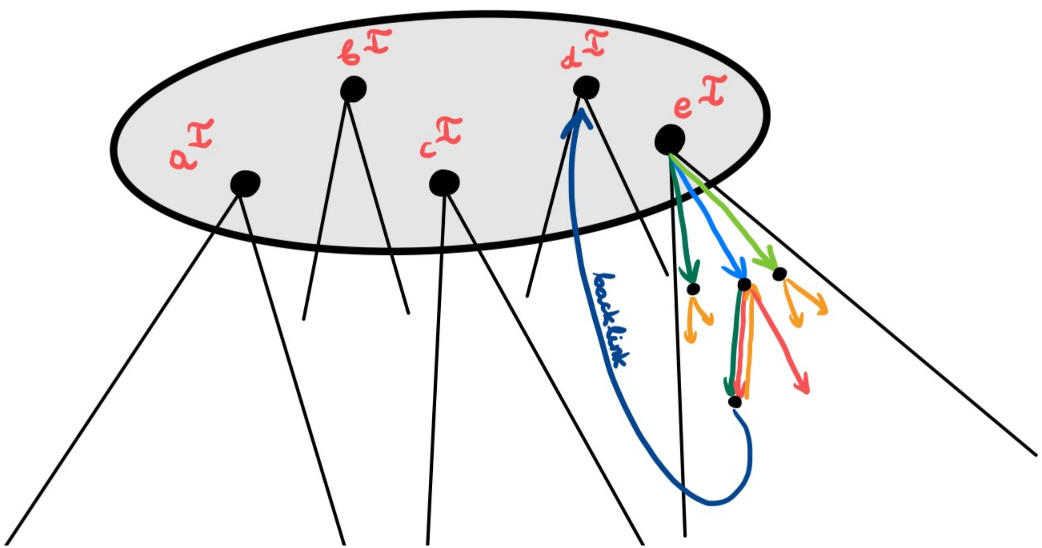

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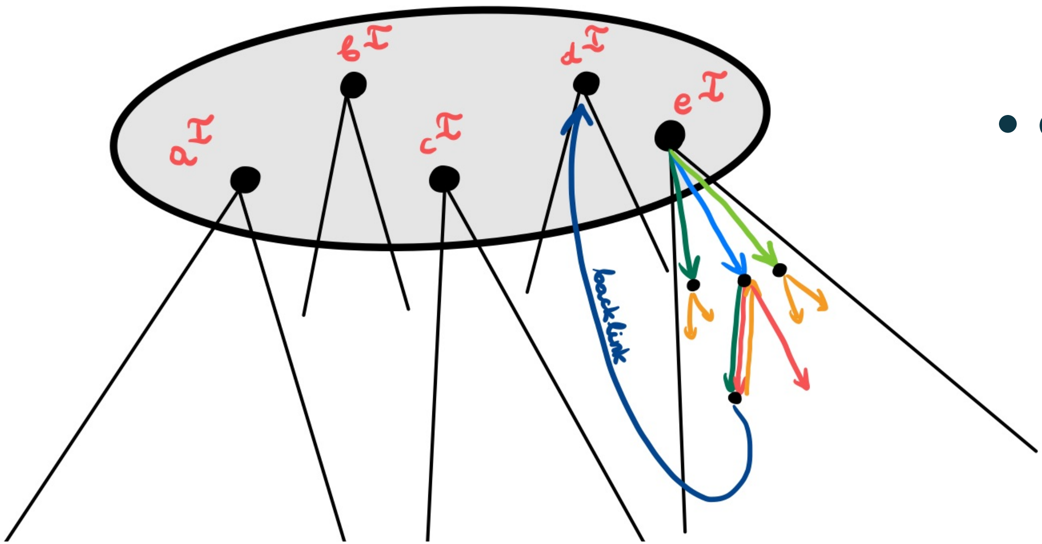

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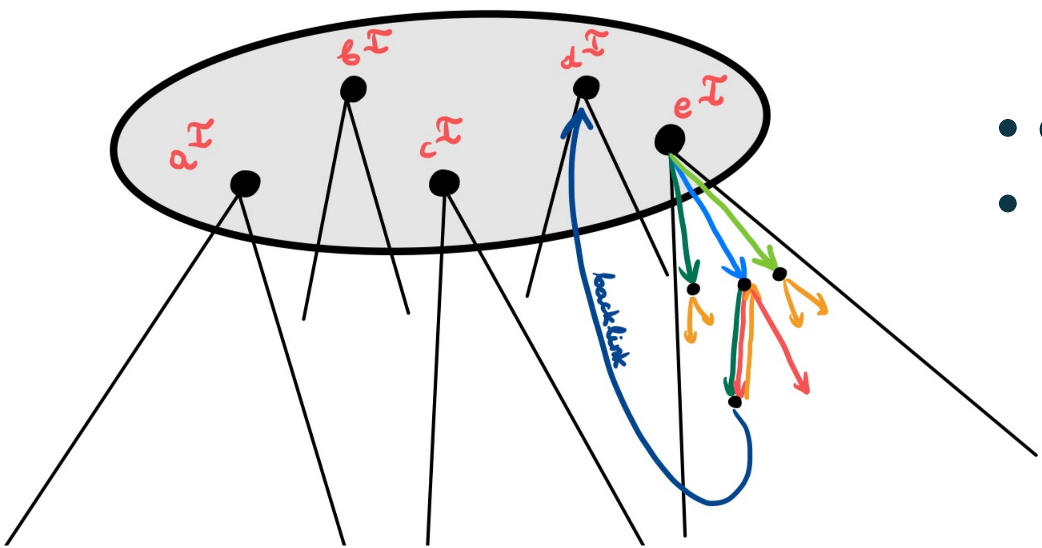

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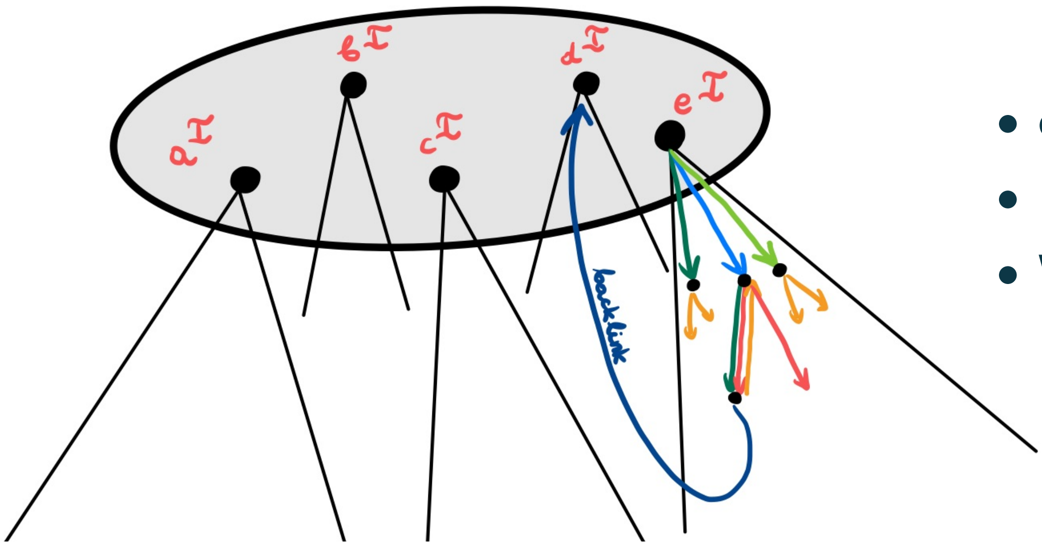

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Atomic type of g:

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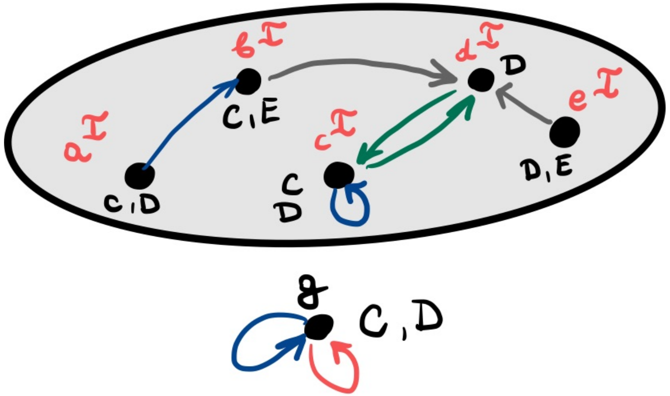

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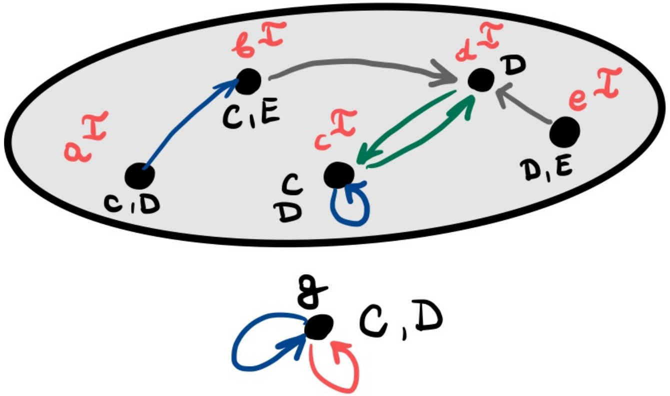

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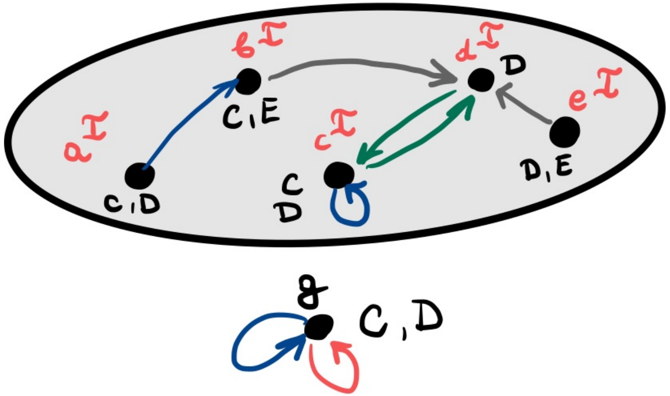

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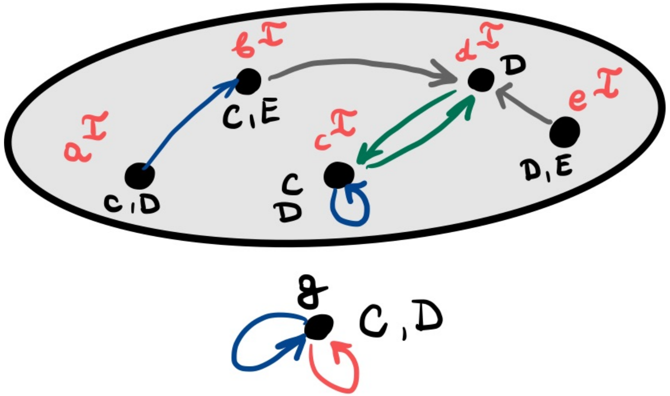

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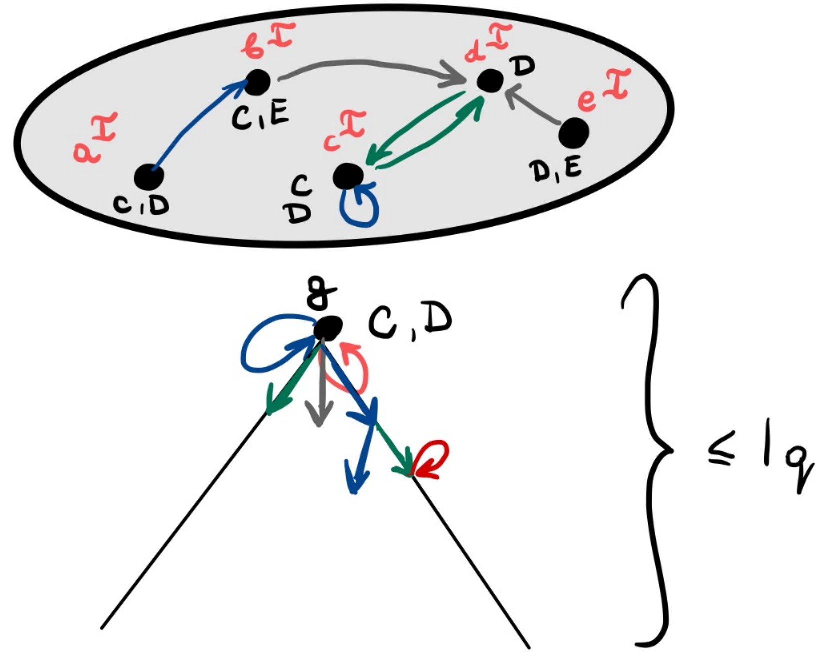

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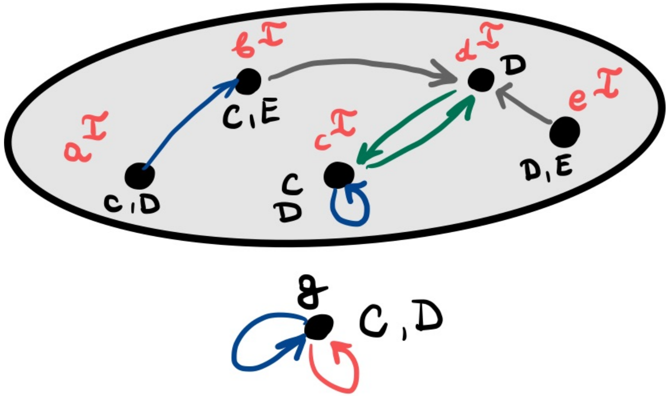

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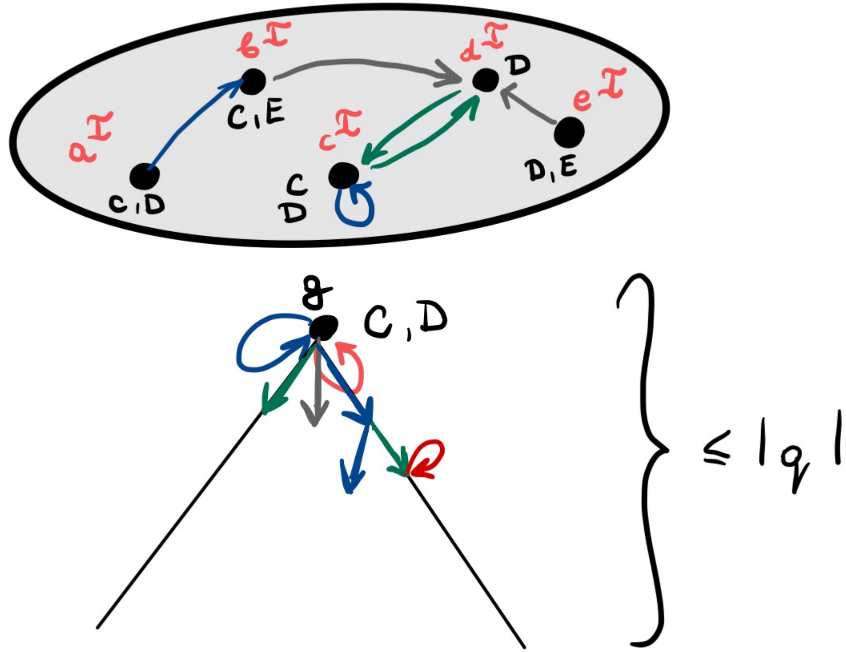

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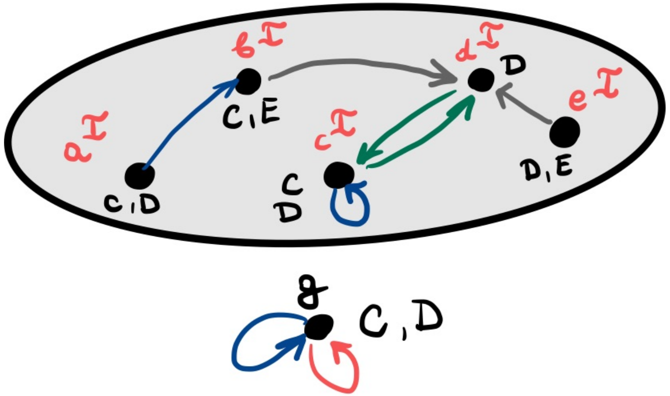

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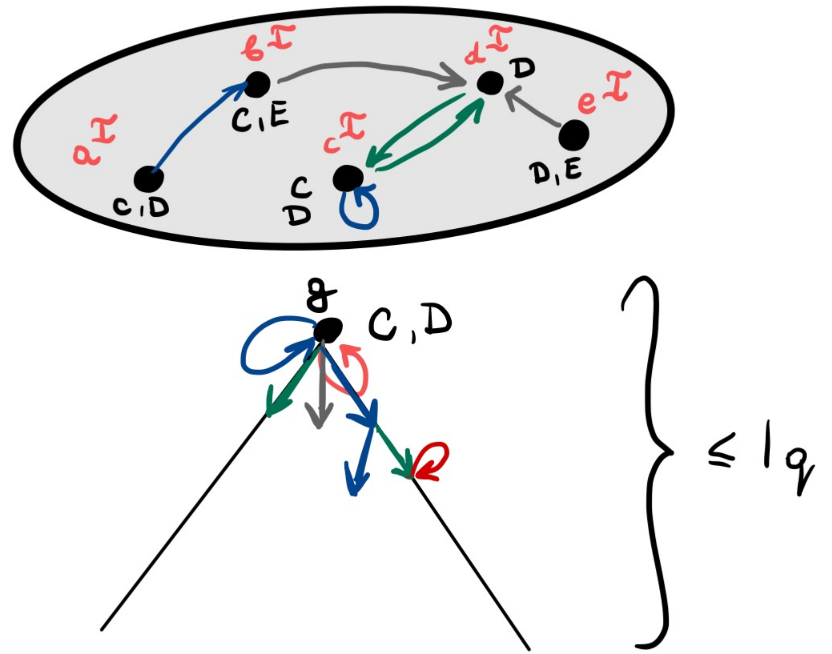

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By finite degree, we know that the set of all downward types $\mathit{DTP}_{\mathcal{I}}$ is finite.

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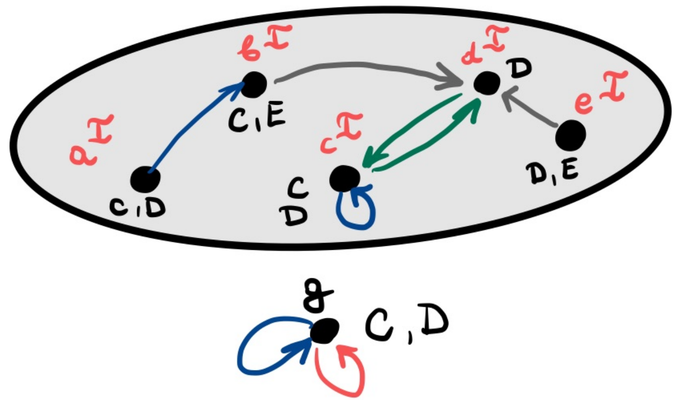
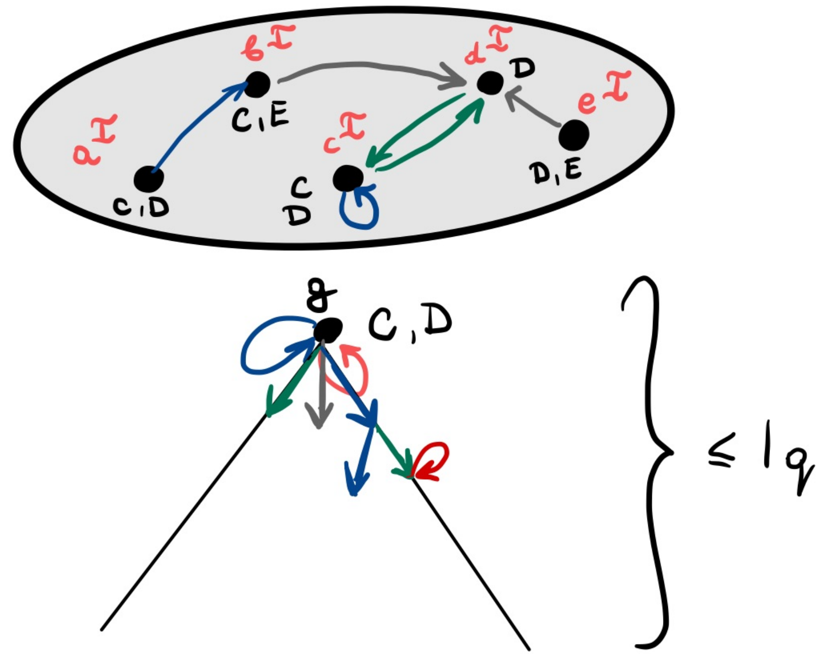


$=$ isomorphism type of $\mathcal{I}\restriction(\{\mathrm{g}\} \cup \mathsf{Nom}_{\mathcal{I}})$

Downward type of g:



$=$ isomorphism type of $\mathcal{I}\restriction(\mathit{Subtree}_{\mathcal{I}}^{\leq|q|}(\mathrm{g}) \cup \mathsf{Nom}_{\mathcal{I}})$

By finite degree, we know that the set of all downward types $DTP_{\mathcal{I}}$ is finite.

We also distinguish the set of nominal downward types $NTP_{\mathcal{I}}$.

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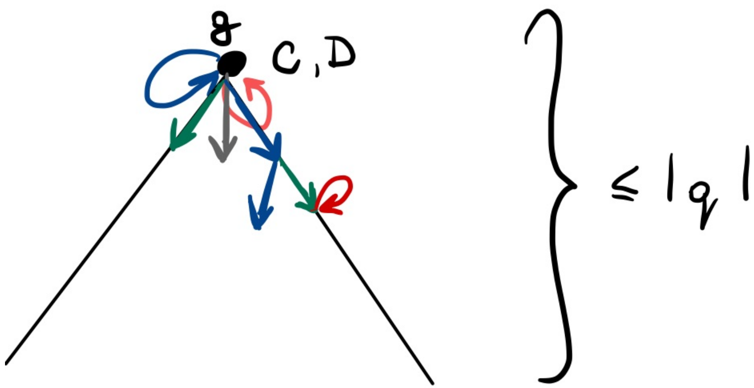

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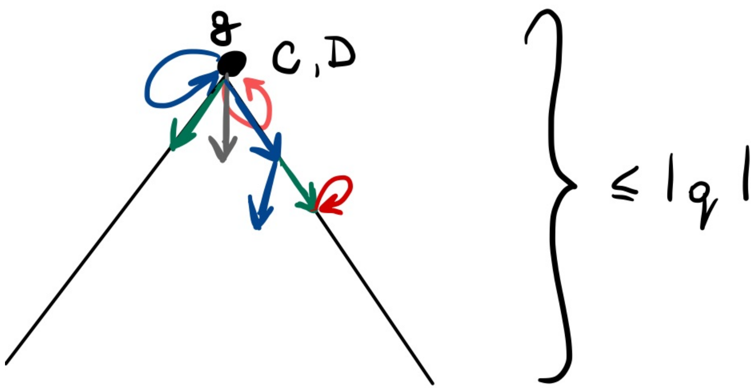

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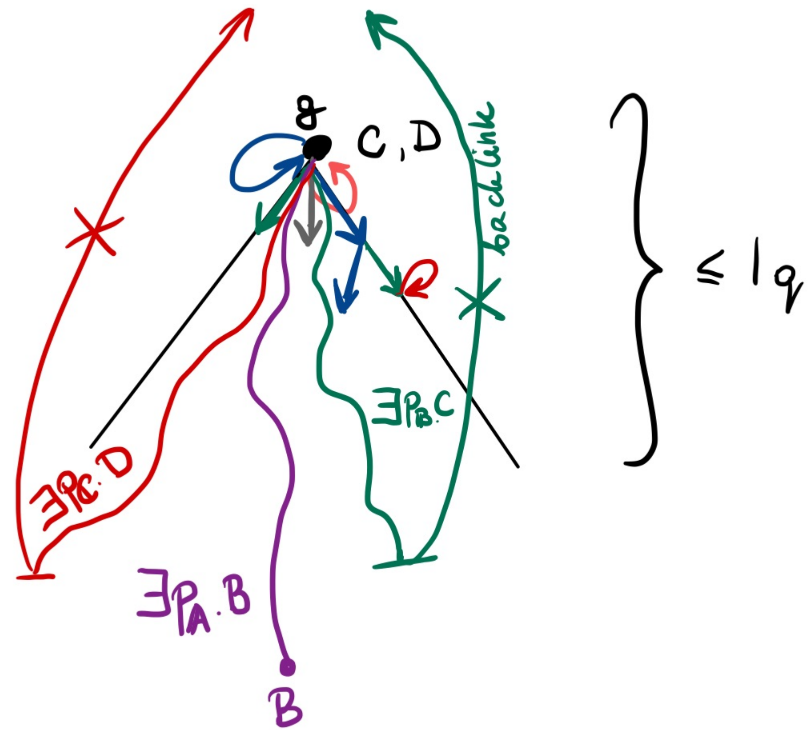

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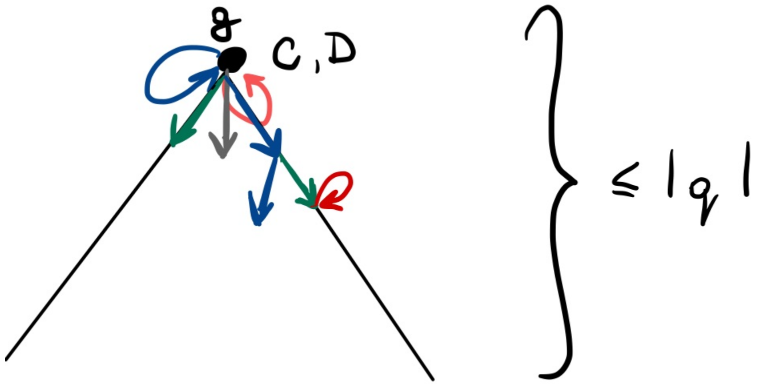

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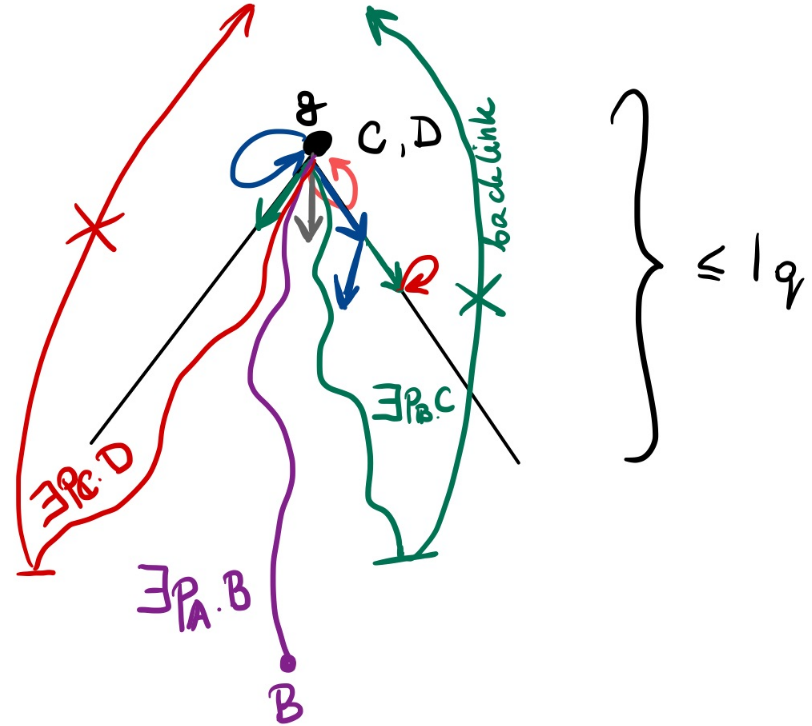

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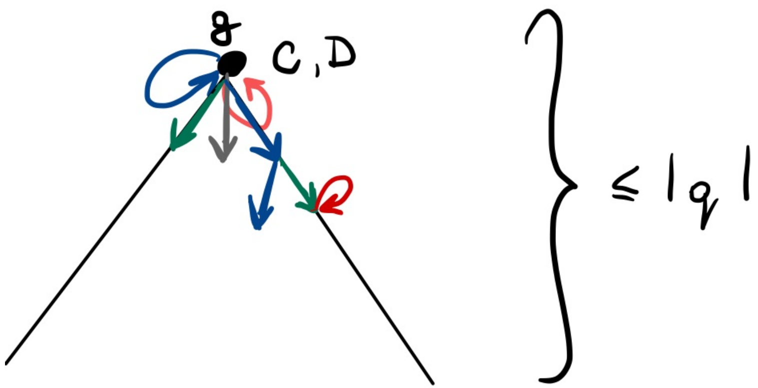

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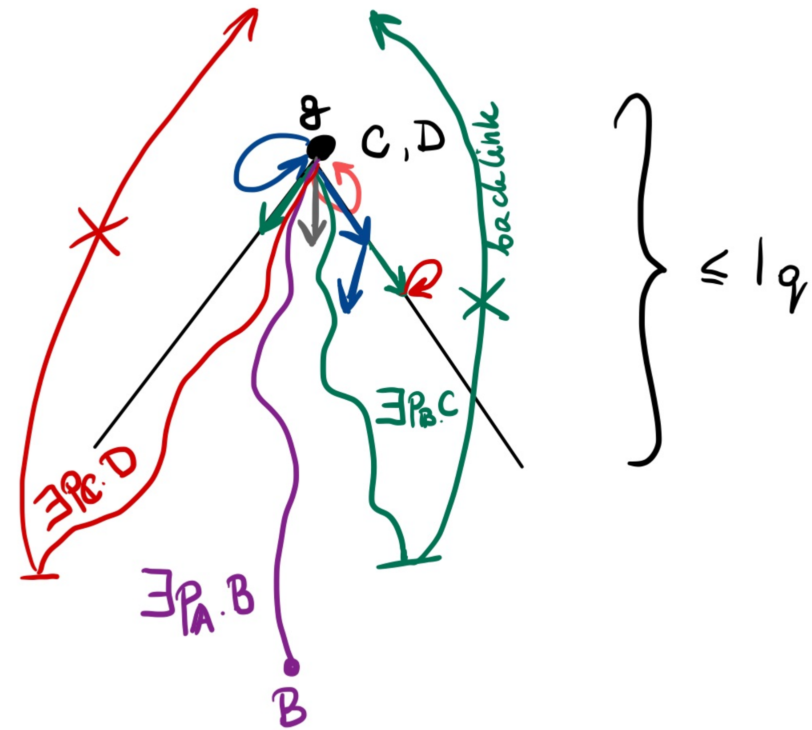

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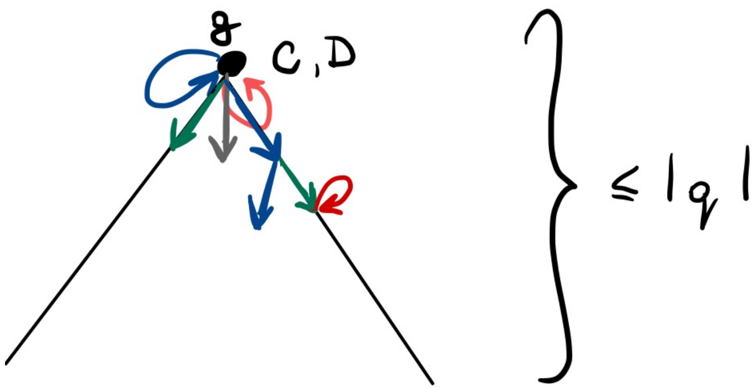

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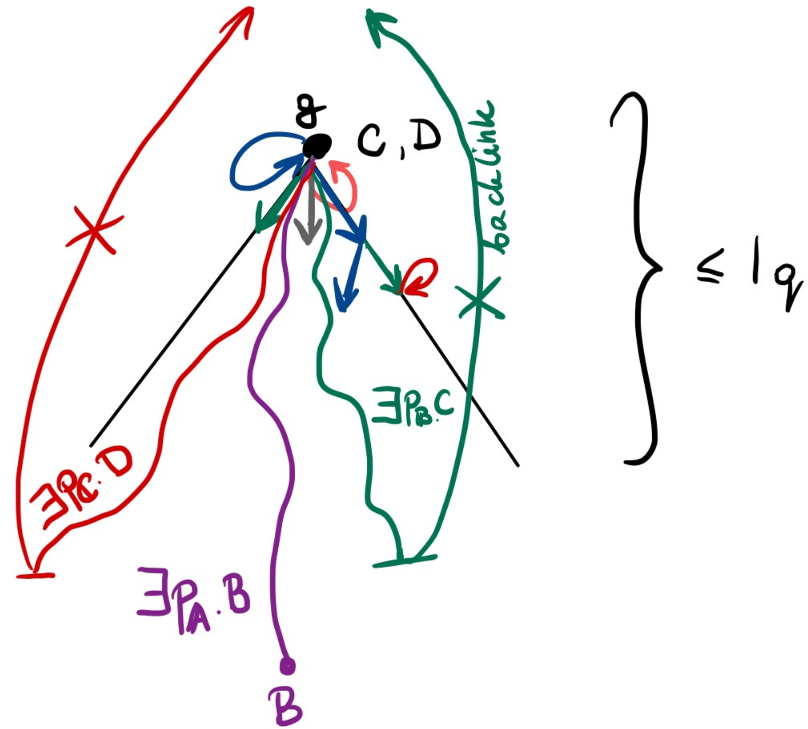

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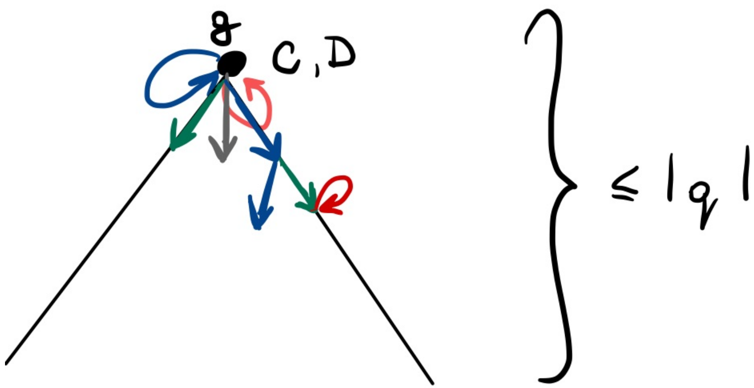

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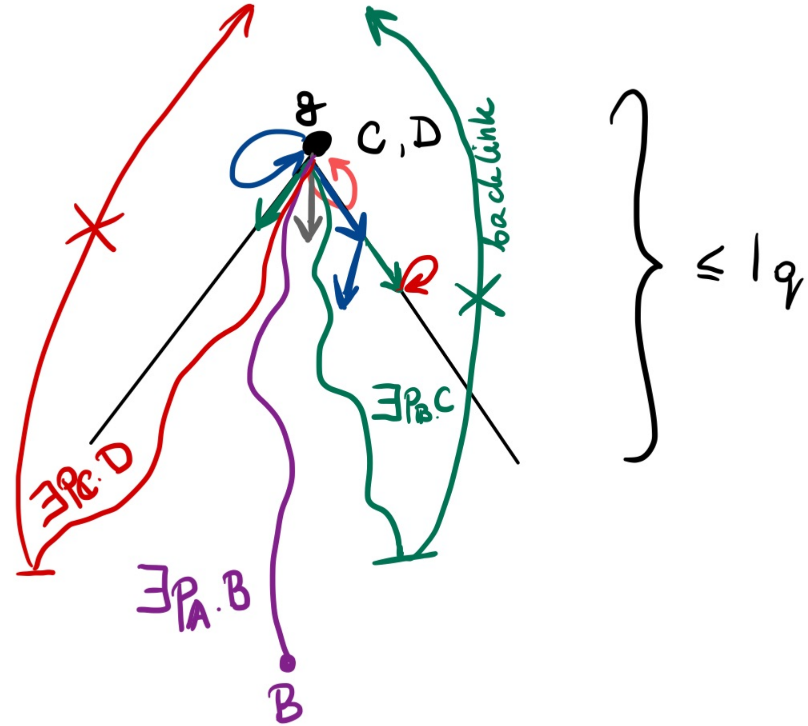

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We extend the resulting structure in a minimal way to make it parent and sibling closed.

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shade 0/1

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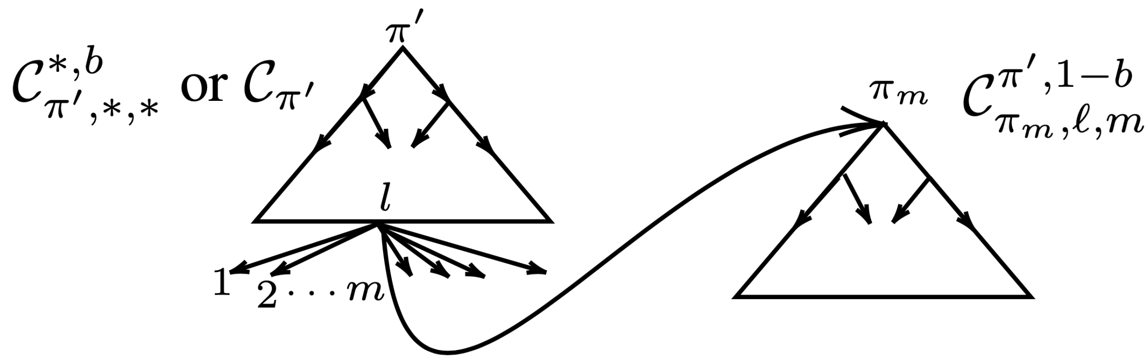

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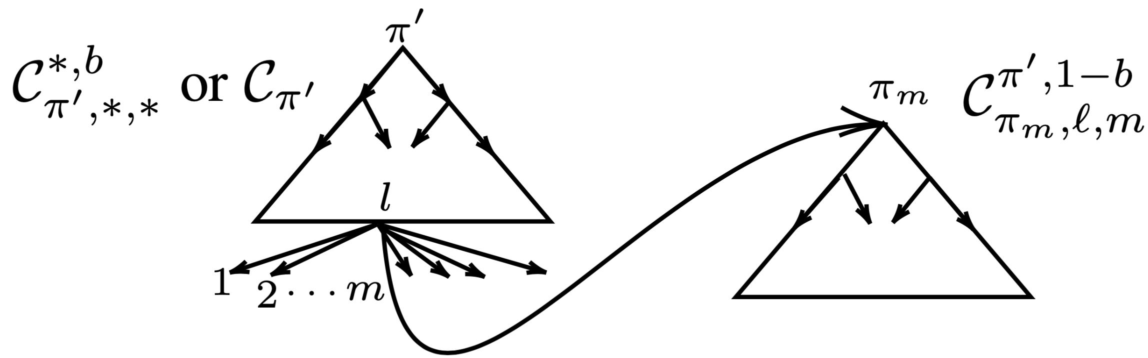

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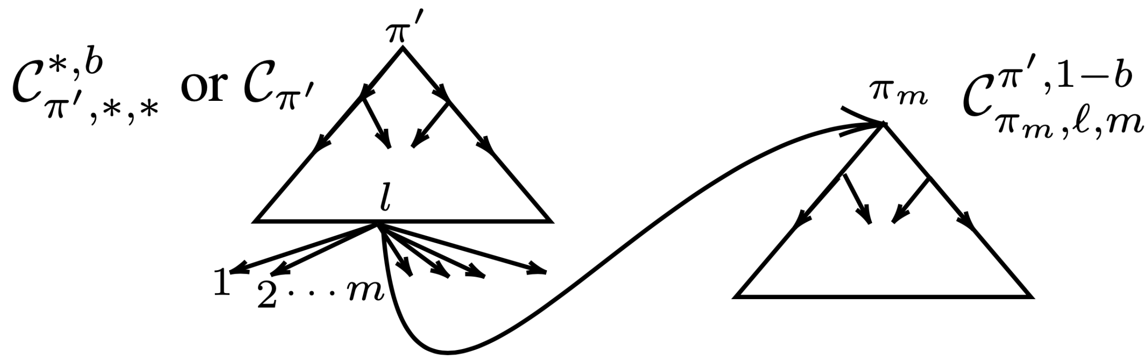

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Check Part IV: The proof in our paper! Thanks for attention!