The Price of Selfishness
Conjunctive Query Entailment for $\mathcal{ALC}_{\mathsf{Self}}$ is 2ExpTime-hard

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Running example: Greek mythology $\mathcal{ALC}_{\mathsf{Self}}$ knowledge base

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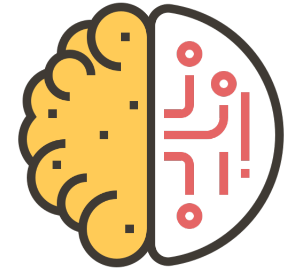

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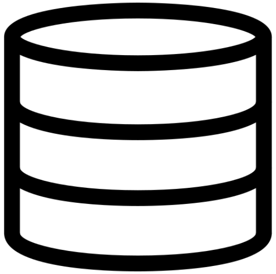

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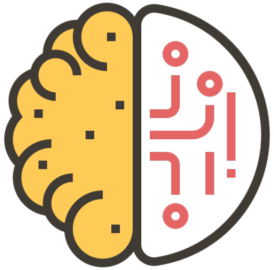

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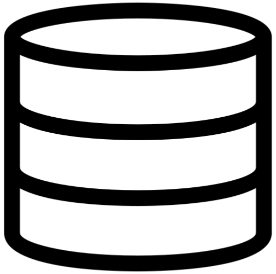

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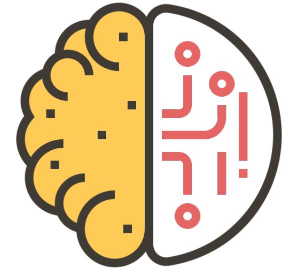

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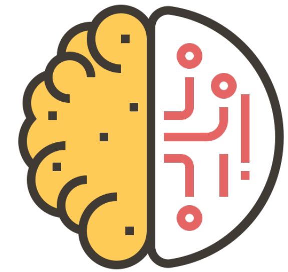

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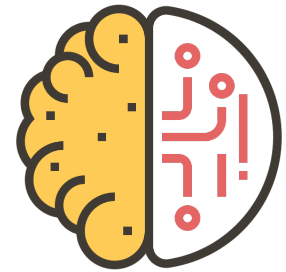

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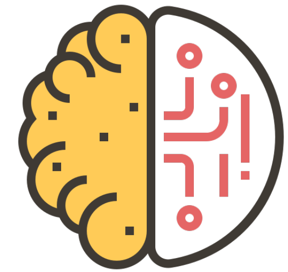

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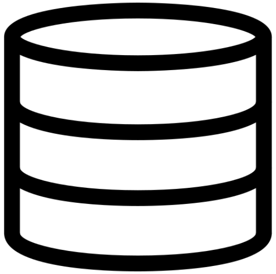

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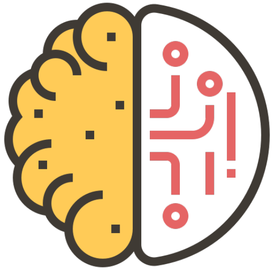

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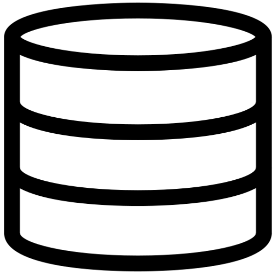

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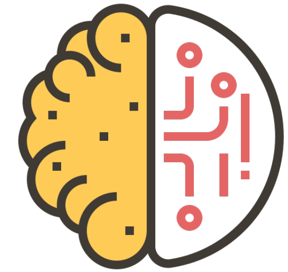

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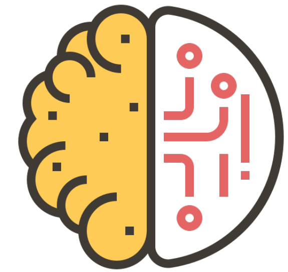

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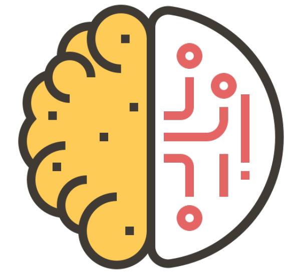

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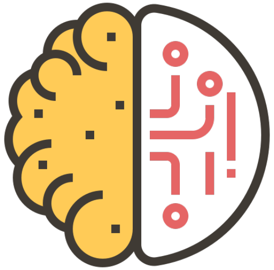

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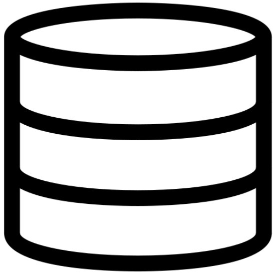

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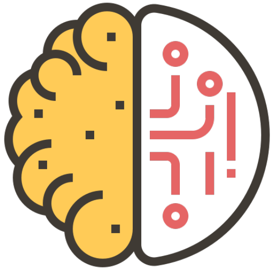

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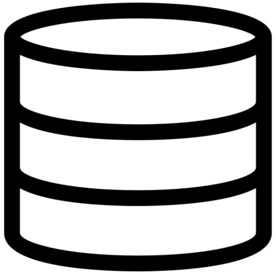

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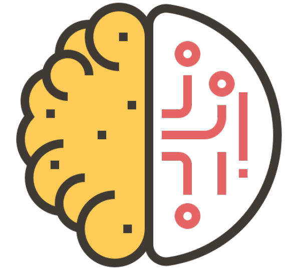

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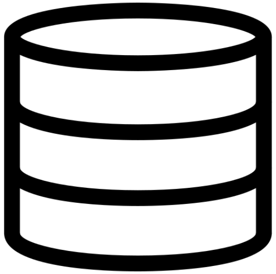

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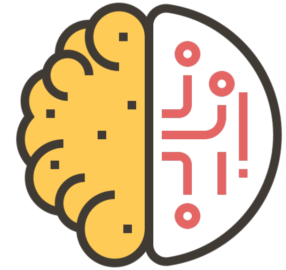

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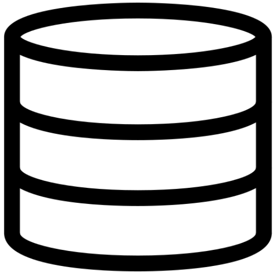

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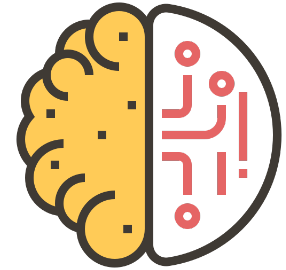

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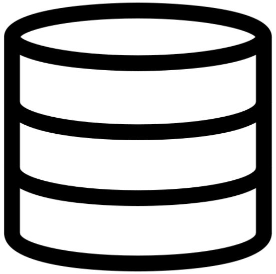

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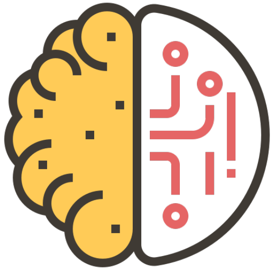

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A knowledge base $\mathcal{K}$ entails a conjunctive query q (written: $\mathcal{K} \models q$) if q matches all models of $\mathcal{K}$.

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- Self is present in OWL2 EL/RL, without harming tractability [Krötzsch et al, ISWC'08]

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- As well as regular expressions, fixed points, (safe) role combination [B.'21, ArXiV]
- And even a tamed use of higher-arity relations [B.'21, JELIA]

2. Some of them increase the complexity exponentially:

E.g. inverses [Lutz'07], transitivity [Eiter et al.'09], nominals (a.k.a. constants) [Ngo et al.'16]

What about the eponymous Self operator? Is it harmless?

Self is supported by OWL 2 Web Ontology Language, $(\exists r.\mathsf{Self})^{\mathcal{I}} := \{d \mid (d, d) \in r^{\mathcal{I}}\}$

- The complexity of satisfiability stays the same, even for very expressive $\mathcal{Z}$ family, a.k.a. $\mathcal{ALCH}b^{\mathsf{Self}}_{\mathsf{reg}}$
- Easy to accommodate in the automata-based approach
- Self is present in OWL2 EL/RL, without harming tractability [Krötzsch et al, ISWC'08]

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hasMother ⊆ hasPar

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W ss?

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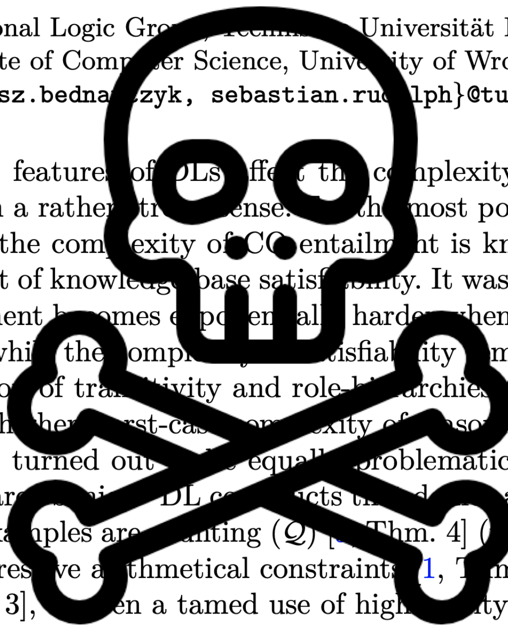
The Price of Selfishness: Conjunctive Query Entailment for $\mathcal{ALC}_{\mathsf{Self}}$ is 2ExpTime-hard

Bartosz Bednarczyk[1,2] and Sebastian Rudolph[1]

[1] Computational Logic Gro Universität Dresden, Germany
[2] Institute of Comp er Science, Unive ity of Wrocław, Poland
{bartosz.bedna zyk, sebastian.ru lph}@tu-dresden.de



Various modelling feature of DLs affe t th complexity of conjunctive query (CQ) entailment in a rather tr ense. most popular basic description logic (DL), $\mathcal{ALC}$, the com exity of CQ entailm nt is known to be ExpTime-complete, as is that of knowledg base satisf ty. It was first shown in [9, Thm. 2] that CQ entailment b omes e po en al harder hen $\mathcal{ALC}$ is extended with inverse roles ($\mathcal{I}$), whi the omp isfiab ity mains the same. Shortly after, a combinatio of tra tivity and role- archie ($\mathcal{SH}$) was shown to be another culprit of h her rst-ca o lexity of easo ng [5, Thm. 1]. Finally, also nominals ($\mathcal{O}$) turned out e equall roblematic [10, Thm. 9]. On the other hand, there ar DL co cts t d affect the complexity of CQ entailment. Exa ples are nting ($\mathcal{Q}$) [Thm. 4] (e complexity stays the same even for expres ve a thmetical constraints 1, T m. 21]), role-hierarchies alone ($\mathcal{H}$) [6, Cor. 3], en a tamed use of hig ty relations [2, Thm. 20].

Conjunctive query entailment over $\mathcal{ALC}_{\mathsf{Self}}$ TBoxes is 2ExpTime-hard.

The skull icon by ©Freepik from flaticon.com.

Conjunctive query entailment over $\mathcal{ALC}_{\mathsf{Self}}$ TBoxes is $2\textsc{ExpTime}$-hard.

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* Hardness does not follow from $\mathcal{SH}$ (no transitivity in CQs!).

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† $\forall x_1\, (\mathrm{self_r}(x_1) \rightarrow \exists x_2 [\mathrm{R}(x_1, x_2) \wedge x_1{=}x_2]) \;\wedge\; \forall x_1 \forall x_2\, (\mathrm{R}(x_1, x_2) \rightarrow [x_1{=}x_2 \rightarrow \mathrm{self_r}(x_2)])$

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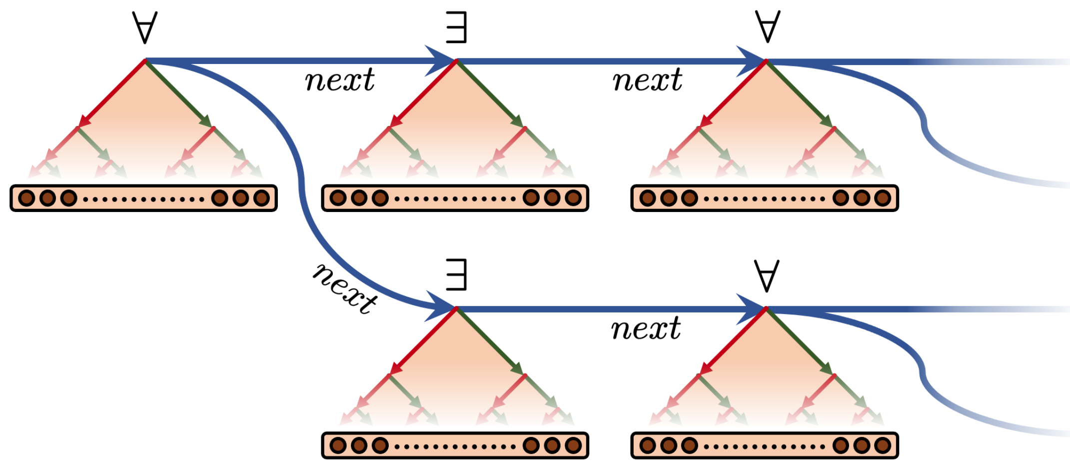

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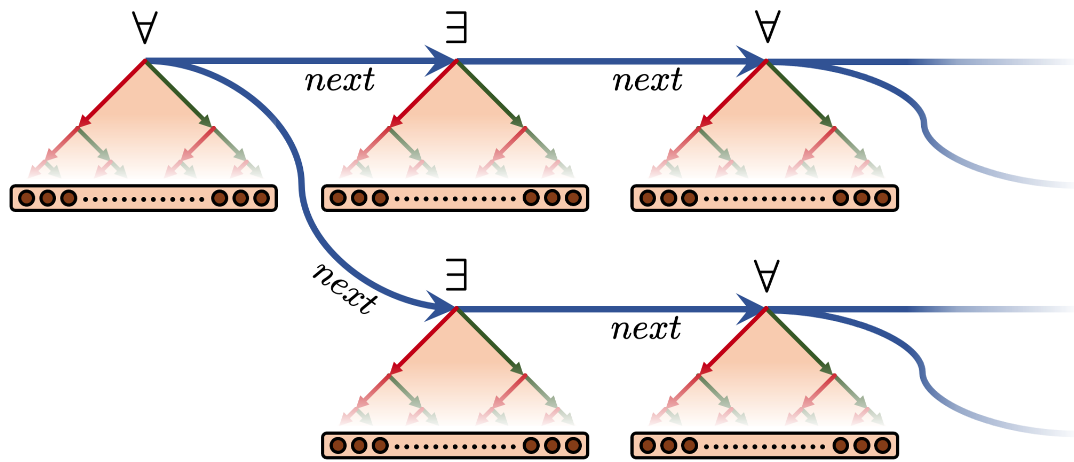

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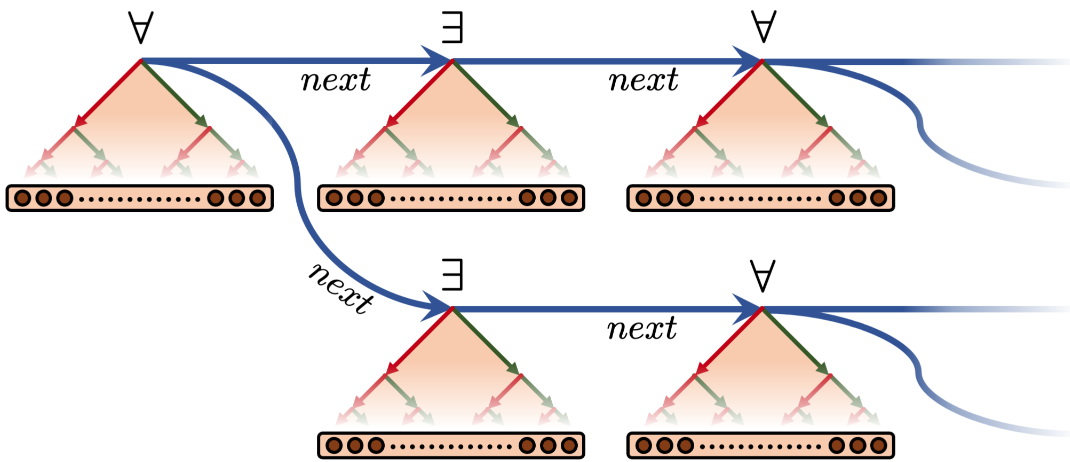

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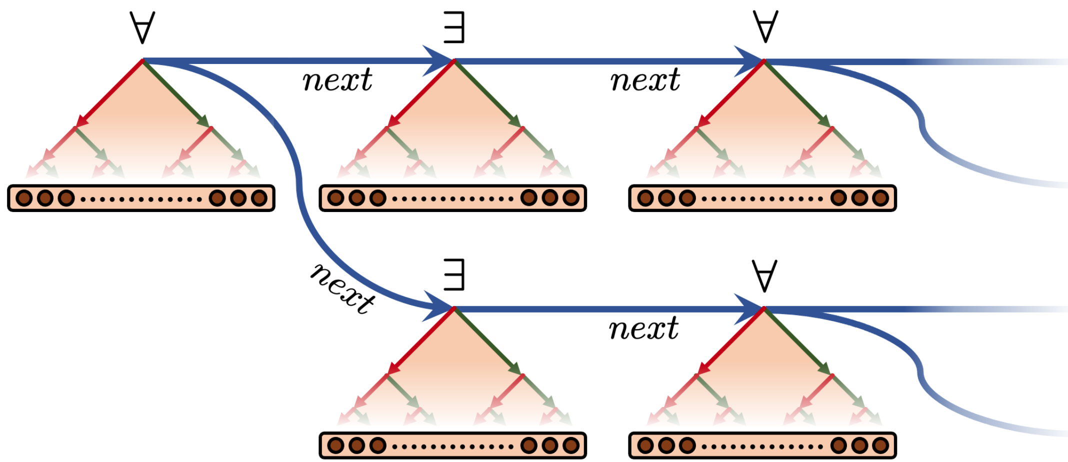

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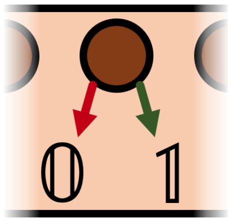

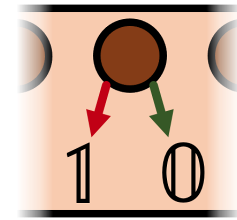

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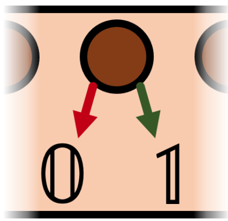

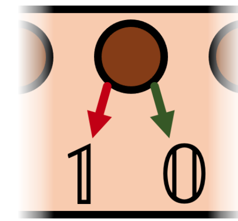

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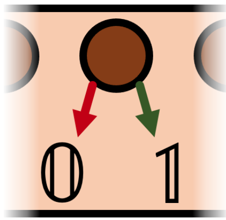

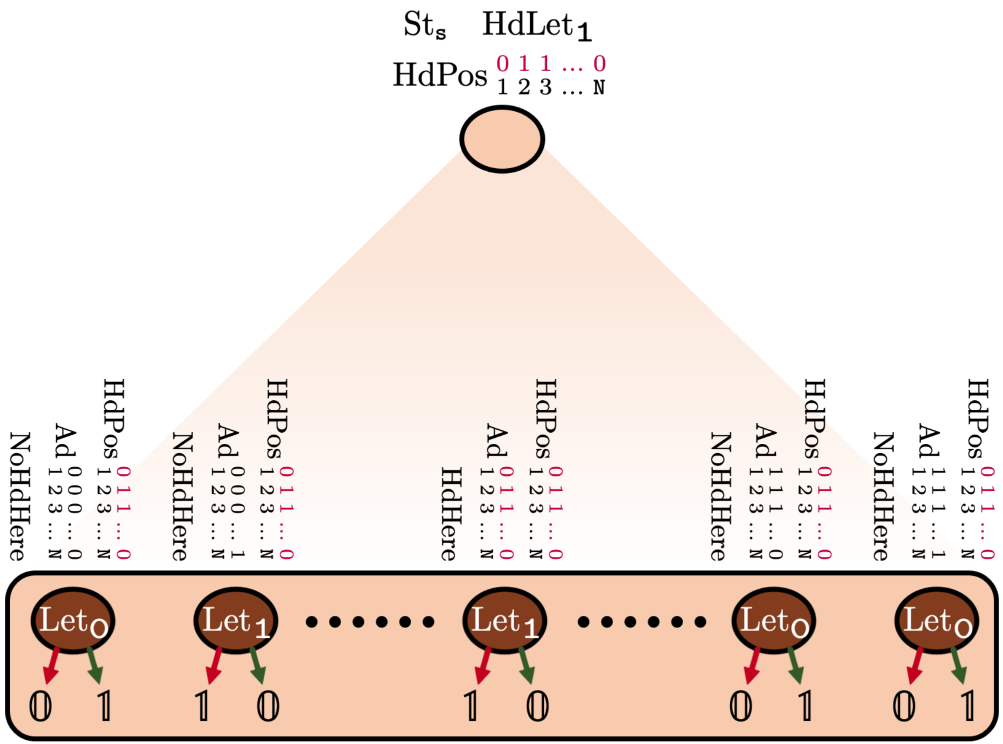

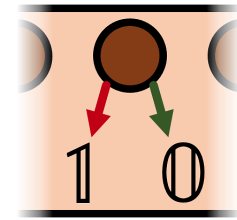

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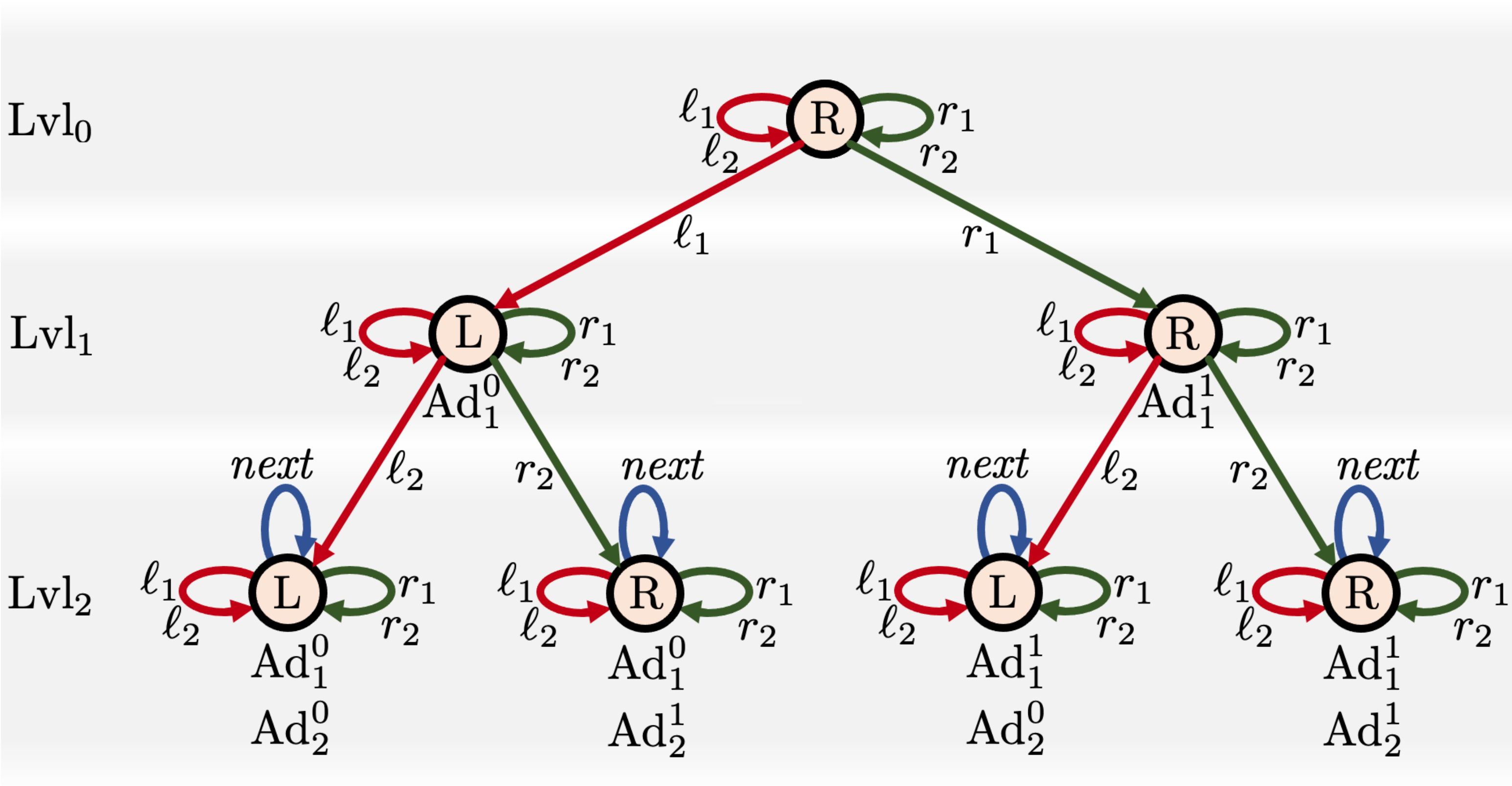
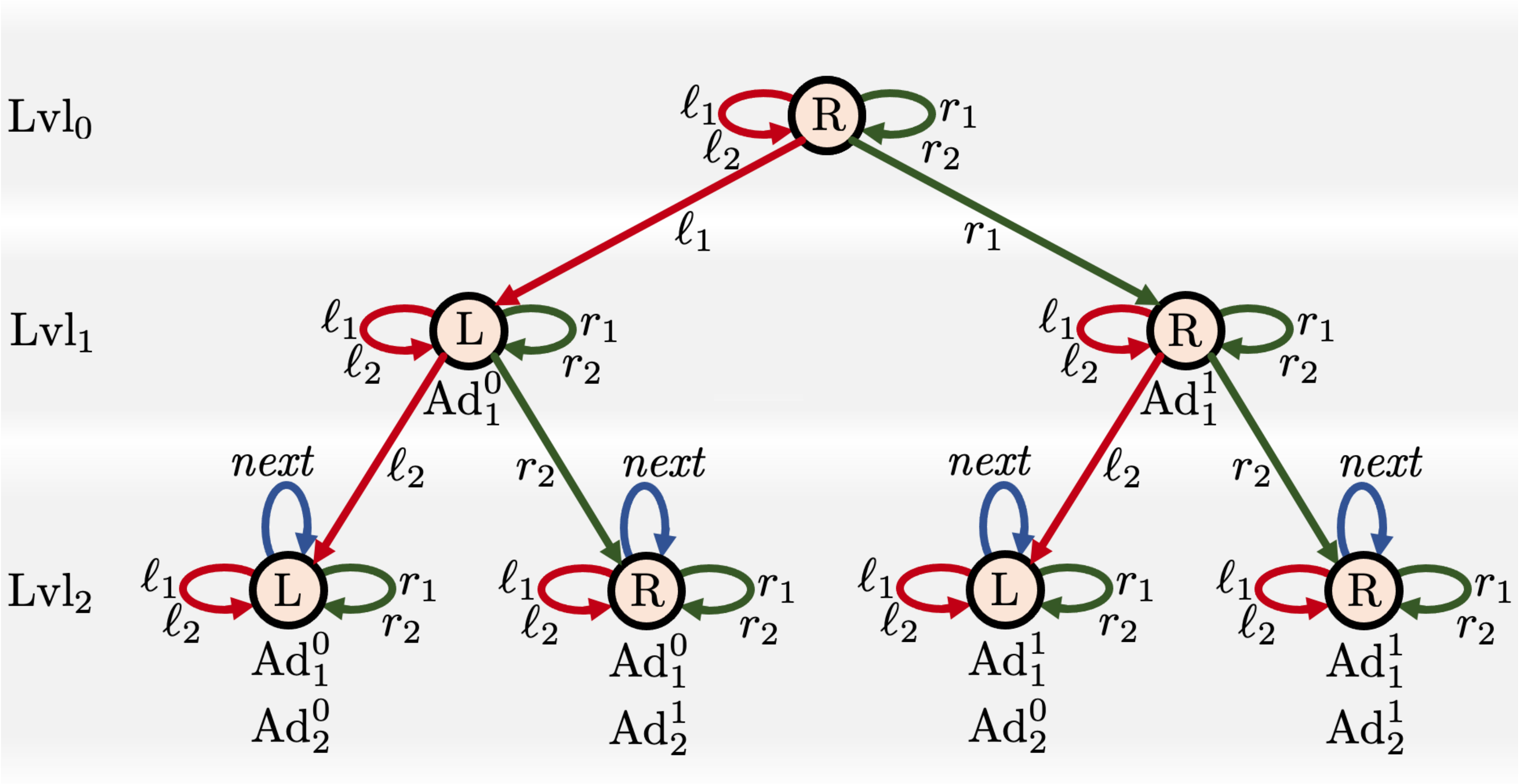


Trick no. 1: A single root-to-leaves conjunctive query

Goal: Design a CQ $q(x, y)$ such that x matches the root and y matches any of the leaves.



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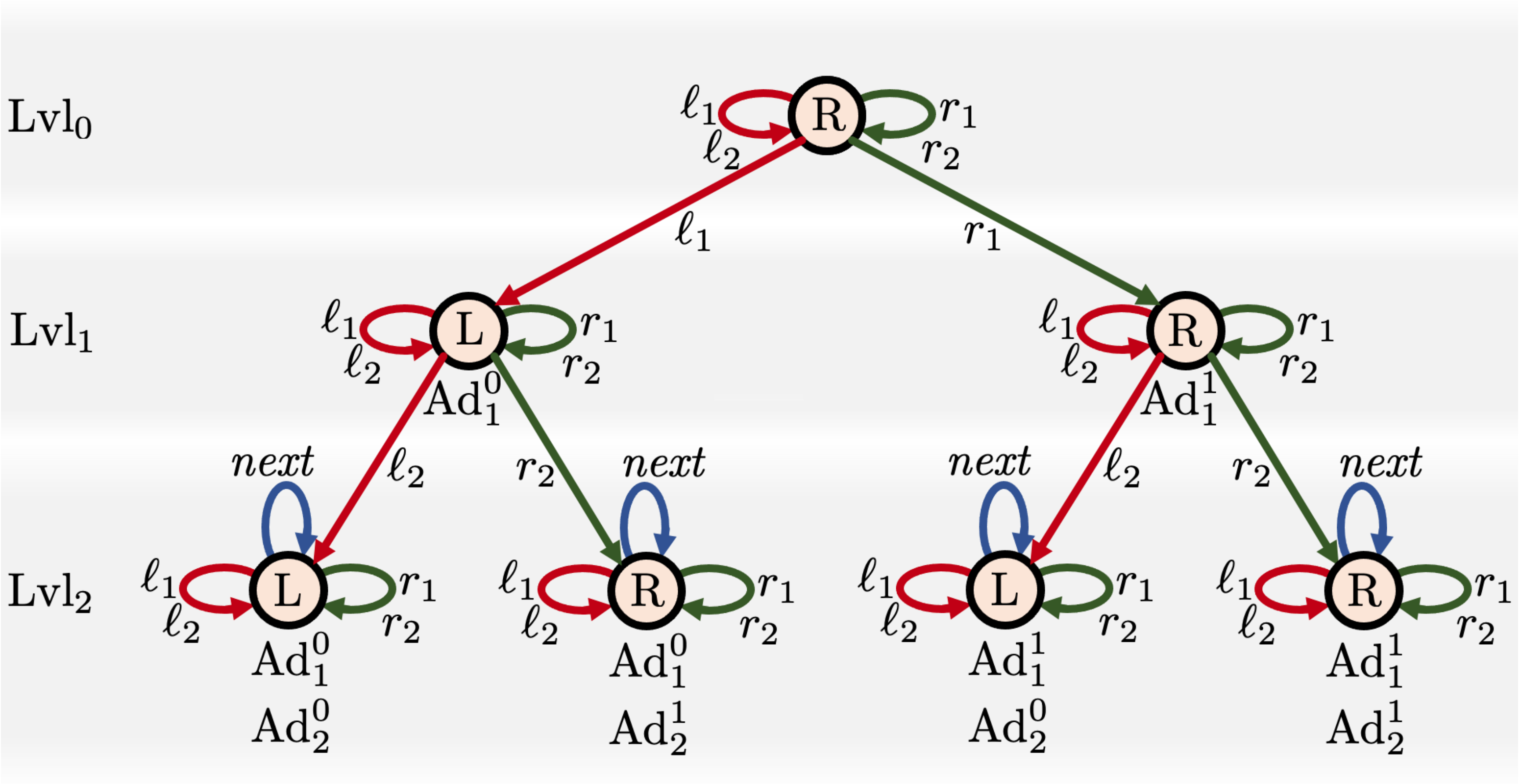

$$\exists x_1 \exists x_2 \exists x_3 \; Lvl_0(x) \wedge \ell_1(x, x_1) \wedge \mathrm{r}_1(x_1, x_2) \wedge \ell_2(x_2, x_3) \wedge \mathrm{r}_2(x_3, y) \wedge Lvl_2(y)$$

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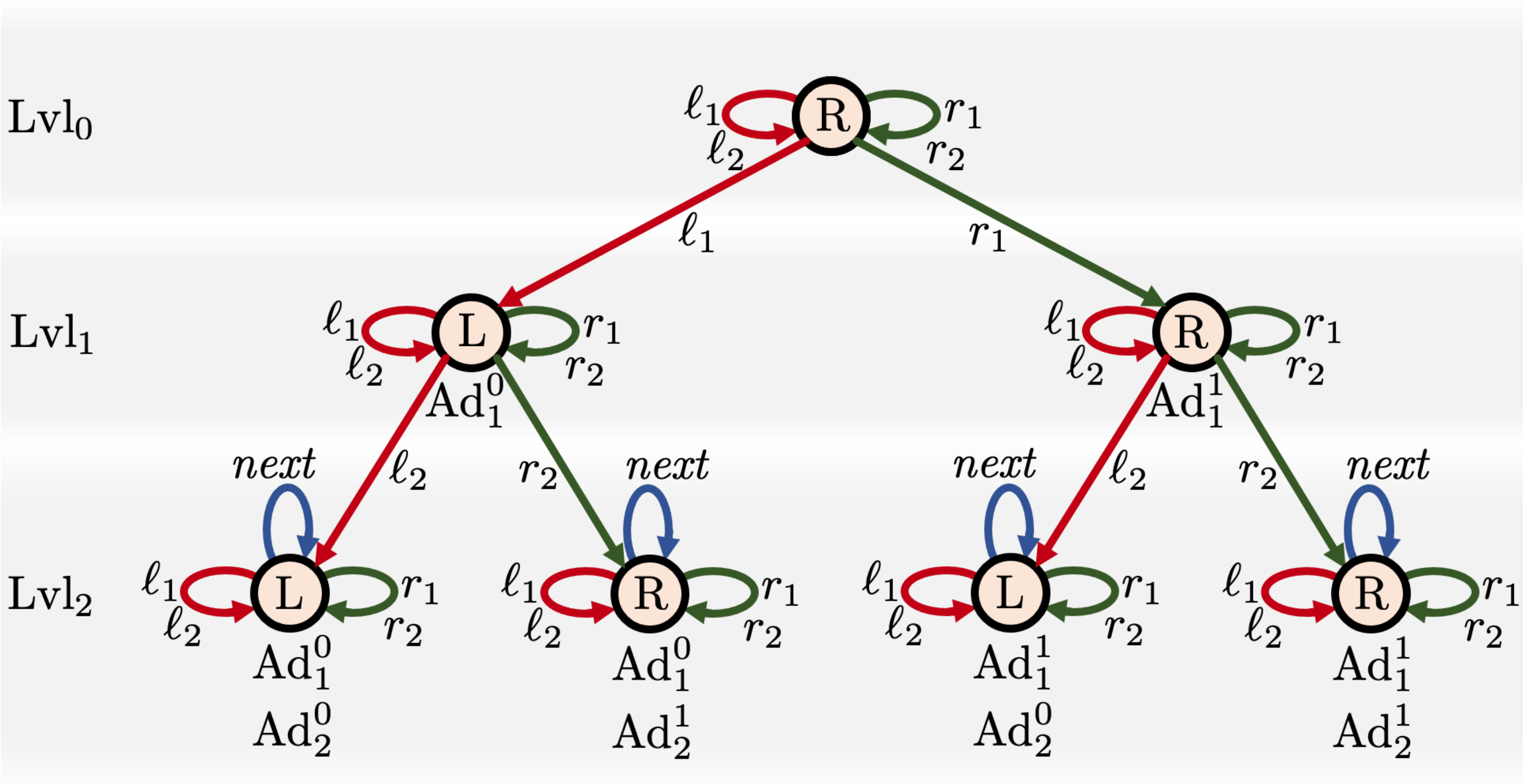

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For brevity we write: $(Lvl_0?; \ell_1; \mathrm{r}_1; \ell_2; \mathrm{r}_2; Lvl_2?)(x, y)$.

Trick no. 2: Synchronisation of leaves among two trees

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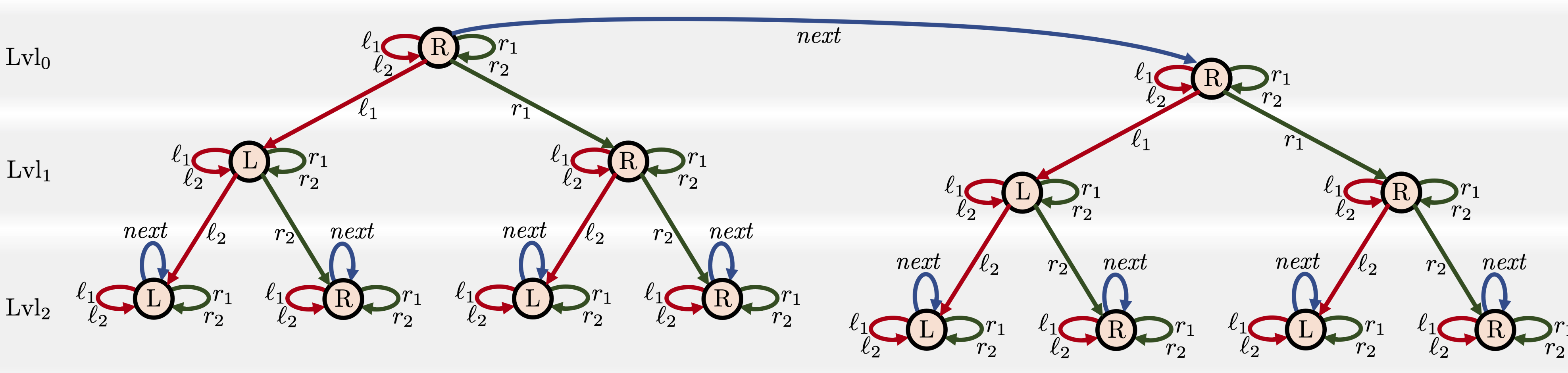

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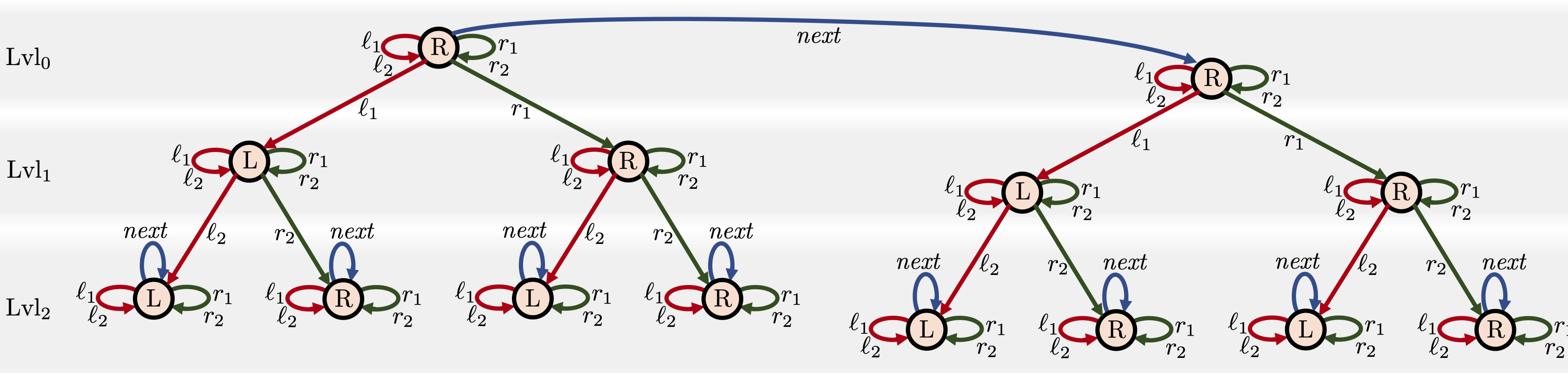
Goal: Design a CQ $q(x, y)$ that matches leaves x, y with equal addresses.



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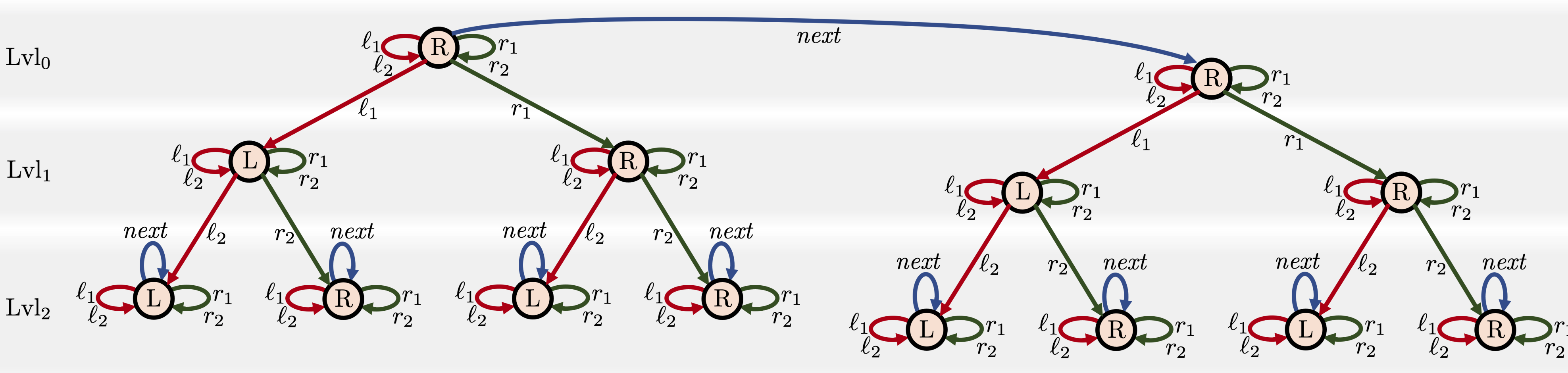

Select two leaves located in different trees:

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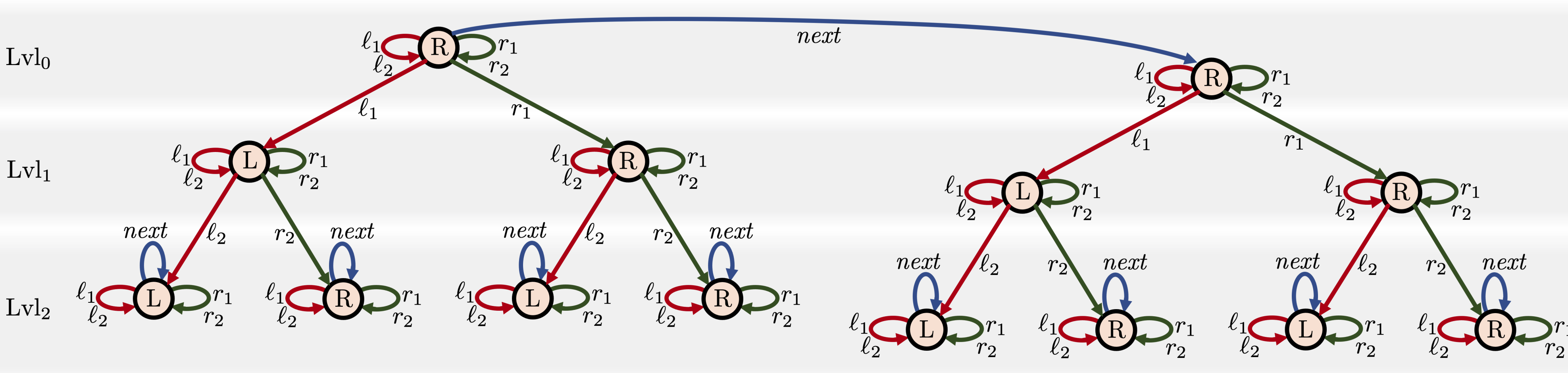

Select two leaves located in different trees:

$$(\mathrm{Lvl}_2?; r_2^-; \ell_2^-; r_1^-; \ell_1^-; \mathrm{Lvl}_0?; \mathit{next}; \mathrm{Lvl}_0?; \ell_1; r_1; \ell_2; r_2; \mathrm{Lvl}_2?)(x, y)$$

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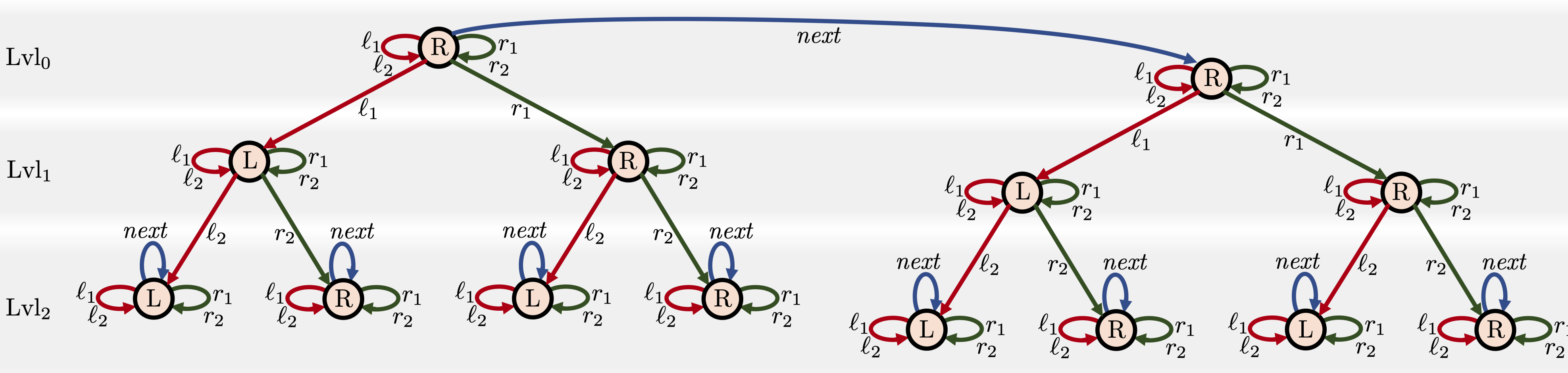

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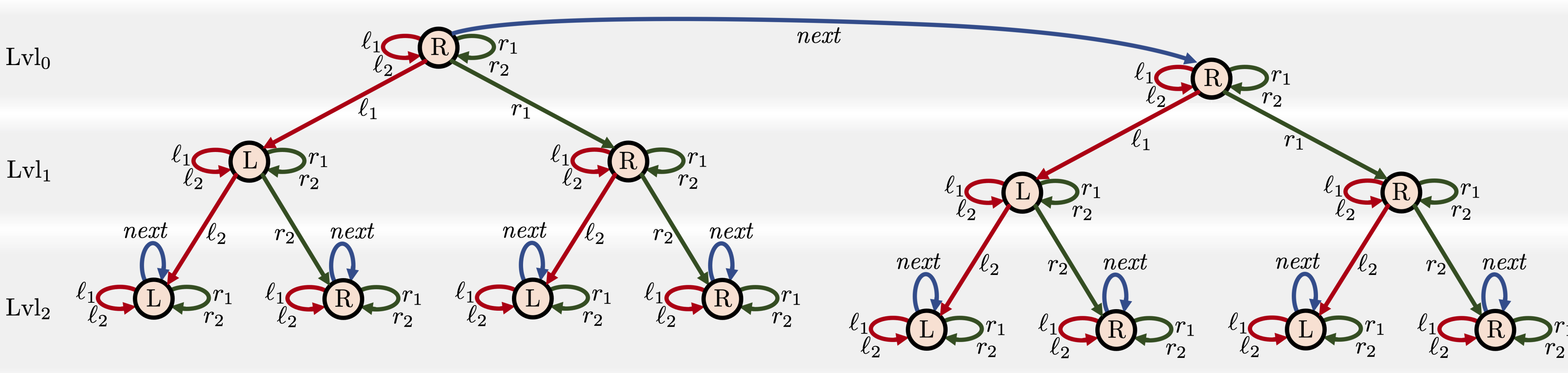

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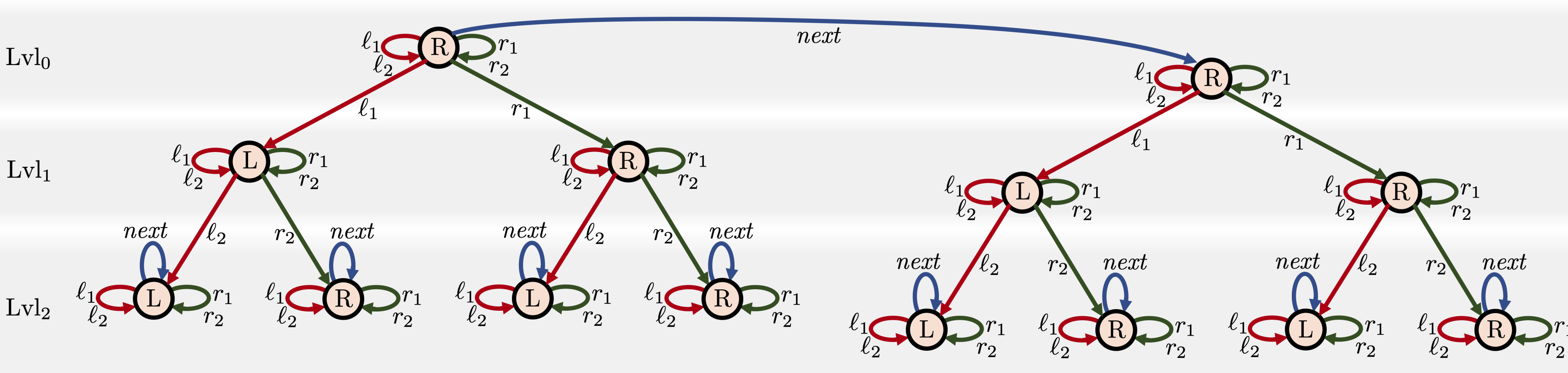

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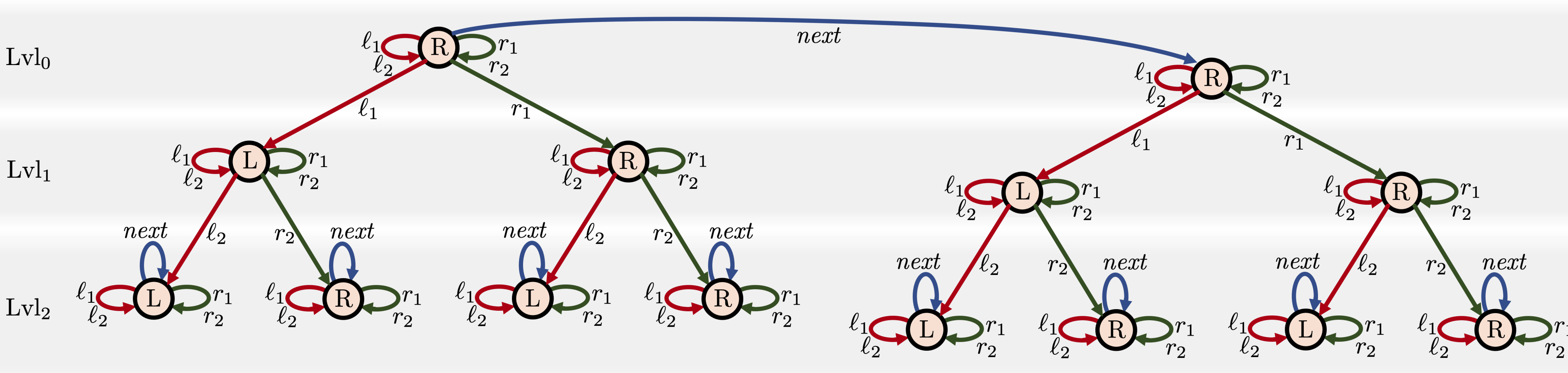

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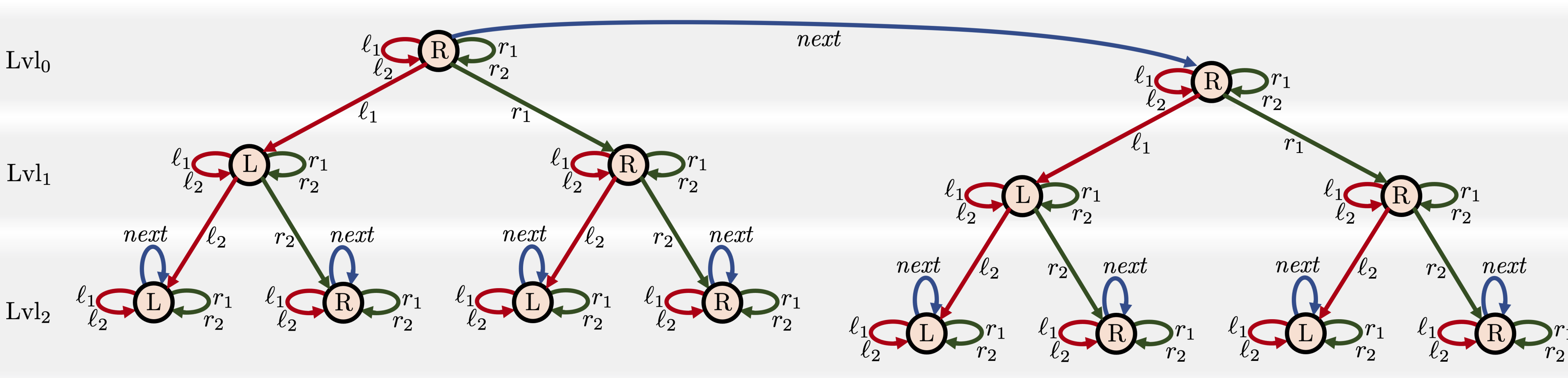
The end: Thanks for your attention!

Biggest challenge: Design a CQ $q(x, y)$ that matches leaves x, y with equal addresses.



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